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Estimation of Gravity Field Parameters by a Multiple Input/Output System

V. D. Andritsanos and I. N. Tziavos

Department of Geodesy and Surveying, Aristotle University of Thessaloniki, 540 06 Thessaloniki, Greece

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Abstract. The recovery of gravity field parameters using various heterogeneous data is performed according to the input/output system theory (IOST) method. The combination of different data sets is carried out by the application of a multiple input - multiple output system. The theory of the algorithm is presented and some conclusions on the assumptions made for the data properties are drawn. Comparisons between a combined system and individual uncorrelated systems are made and the proper use of the data sets in each case is discussed. Finally, an application is presented, where input data, such as shipborne gravity anomalies and sea surface heights (SSHs) derived from different satellite missions, are optimally combined in order to estimate marine geoid heights and sea surface topography (SST).

1 Introduction

The use of spectral techniques in gravity field modeling has been developed over the past decade. Spectral methods have been used for the efficient evaluation of convolution integrals in terrain correction formulas, e.g., Sideris (1984); Li (1993), in geoid determination, e.g., Sideris and Li (1993); Forsberg and Sideris (1993); Strang van Hees (1990); Haagmans et al. (1993); Tziavos (1993); Tziavos (1995), in deflection of the vertical computations, e.g., Kearsley et al. (1985); Liu et al. (1997); Tziavos and Andritsanos (1998) and in Molodensky's problem solution, e.g., Sideris (1987). The use of spectral techniques in physical geodesy is summarized in Schwarz et al. (1990). Nevertheless, the noiseless data and the data homogeneity assumptions were their main drawbacks in geodetic applications. Recently, the use of heterogeneous noisy data in spectral gravity field modeling was presented by Sideris (1996) based on the input/output system theory (Bendat and Piersol, 1986). Simulation studies were carried out by Li (1996); Tziavos et al. (1996a); Tziavos et al. (1996b); Tziavos et al. (1996c) and an application to airborne gravity is described by Wu and Sideris (1995). The similarities and differences between systems theory and least-squares collocation are analyzed in Sansò and Sideris (1997). In this paper, the theory of a multiple input/ multiple output system in gravity field parameter estimation is briefly presented. Comparisons between a generalized algorithm and the individual systems algorithm are carried out and some schemes related to the proper filtering of the data and the estimation of the output parameters are presented.

Correspondence to: V. D. Andritsanos

2 Theoretical background

2.1 Multiple input/output system description

A multiple input/output system with noise is presented in figure 1, where $x_1, x_2, \ldots, x_q$ are the q input signals, $m_1, m_2, \ldots, m_q$ are the input signal noises and $x_{1o}, x_{2o}, \ldots, x_{qo}$ are the observed data. The transfer functions (frequency response functions) $h_{1o}, h_{2o}, \ldots, h_{qo}$ of the system represent the modified original transfer functions in order to filter-out the noise. The output signals are contained in vector y and the output noises in n. A generalized formulation of the

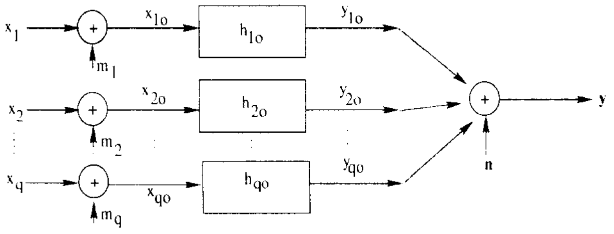

Fig. 1. Multiple input/output system with noise

above system can be obtained by writing the data structure in vector and matrix notations. The case of q input signals and w output signals can be formulated as follows:

$$\mathbf{X_o} = \begin{bmatrix} X_1 + M_1 \\ X_2 + M_2 \\ \vdots \\ X_q + M_q \end{bmatrix} \qquad \mathbf{Y_o} = \begin{bmatrix} Y_1 - N_1 \\ Y_2 - N_2 \\ \vdots \\ Y_w - N_w \end{bmatrix}, \tag{1}$$

where the Fourier transforms (spectra) are denoted by capital letters. The relation between the input noisy signal and the output is given by the following matrix equation:

$$\mathbf{Y} = \mathbf{H}^{\mathbf{T}}_{\mathbf{x_o y_o}}(\mathbf{X} + \mathbf{M}) + \mathbf{N}, \tag{2}$$

where the transfer function matrix is

$$\mathbf{H_{x_o y_o}} = \begin{bmatrix} H_{x_{1o} y_{1o}} & H_{x_{1o} y_{2o}} & \cdots & H_{x_{1o} y_{wo}} \\ H_{x_{2o} y_{1o}} & H_{x_{2o} y_{2o}} & \cdots & H_{x_{2o} y_{wo}} \\ \vdots & \vdots & \ddots & \vdots \\ H_{x_{qo} y_{1o}} & H_{x_{qo} y_{2o}} & \cdots & H_{x_{qo} y_{wo}} \end{bmatrix}. \tag{3}$$

The final solution is obtained by the minimization of the output error Power Spectral Density (PSD). Following Bendat and Piersol (1980), Bendat and Piersol (1986) and Sideris (1996), the output error PSD matrix is computed using (2):

$$\left\{ \begin{array}{l} \mathbf{N} = \mathbf{Y} - \mathbf{H}^{\mathbf{T}}_{\mathbf{x_o y_o}}(\mathbf{X} + \mathbf{M}) \\ \mathbf{N}^* = \mathbf{Y}^* - \mathbf{H}^{\mathbf{T}*}_{\mathbf{x_o y_o}}(\mathbf{X}^* + \mathbf{M}^*) \end{array} \right\} \Rightarrow$$

$$\begin{aligned} \mathbf{P_{nn}} &= \mathbf{P_{yy}} - \mathbf{P_{yx_o}} \mathbf{H_{x_o y_o}} \\ &- \mathbf{H}^{\mathbf{T}*}_{\mathbf{x_o y_o}} \mathbf{P_{x_o y}} + \mathbf{H}^{\mathbf{T}*}_{\mathbf{x_o y_o}} \mathbf{P_{x_o x_o}} \mathbf{H_{x_o y_o}} \end{aligned} \tag{4}$$

According to the minimization criterion

$$\frac{\partial \mathbf{P_{nn}}}{\partial \mathbf{H}^{\mathbf{T}*}_{\mathbf{x_o y_o}}} = \mathbf{0} \tag{5}$$

the optimal transfer function matrix can be computed by the following relation:

$$\mathbf{H_{x_o y_o}} = \mathbf{P}^{-1}_{\mathbf{x_o x_o}} \mathbf{P_{x_o y}} \tag{6}$$

Assuming no correlation between input signals with the input noises, the optimal transfer function matrix is

$$\mathbf{H_{x_o y_o}} = (\mathbf{P_{xx}} + \mathbf{P_{mm}})^{-1} \mathbf{P_{xy}} \tag{7}$$

The evaluation of the input-output PSD matrix is possible only if the input noise PSD matrix is known. Then, $\mathbf{P_{xy}}$ can be computed by:

$$\mathbf{P_{xy}} = \mathbf{H}^{\mathbf{T}}_{\mathbf{xy}} \mathbf{P_{xx}} = \mathbf{H}^{\mathbf{T}}_{\mathbf{xy}} (\mathbf{P_{x_o x_o}} - \mathbf{P_{mm}}), \tag{8}$$

where $\mathbf{H_{xy}}$ is the transfer function matrix which connect the pure input and output signal. For example, if gravity anomaly is chosen as input signal and geoid as output signal, then, theoretically, $H_{\Delta g N}$ is nothing else but the Stokes operator in the frequency domain.

2.2 Advantages and drawbacks of the method

The multiple input/output system theory is a spectral technique. This fact contributes to the fast and efficient handling of large amounts of data. New heterogeneous data can be combined using this spectral technique. For example, satellite, airborne, marine and terrestrial data can be used for an optimal combination solution. Input errors can be easily propagated into the results. The proper modification of the transfer functions can be achieved in order to minimize the noise-to-signal ratio. In this manner, the input noise is filtered out and error estimates for the predicted results are provided. Multiple input/output system results are efficiently calculated by computer algorithms due to the smaller matrix dimensions in comparison to other space domain techniques, such as least-squares collocation. In addition, all matrix computations are evaluated in the frequency domain with the convenient matrix division (frequency-by-frequency division) rather than the complicated matrix inversion in the space domain.

Nevertheless, some assumptions for the signal and the noise are needed. The fundamental difficulty with the frequency domain solution is that the input error PSD must be known (Sideris, 1996). Only the variances of the measurements are known in practice and not the errors themselves. Since the error variances change from point to point, we are dealing with non-stationary noise. The algebraic simplicity of the solution is lost when the data noise is non-stationary. The simple algebraic relations in the frequency domain become complicated integral equations when the stationarity assumption is eliminated (Sansò and Sideris, 1997) and, therefore, we cannot simply obtain the input error PSD as the Fourier transform of the input error covariance matrix (Sideris, 1996). This problem is discussed in details in Sansò and Sideris (1997). In the next sections, more remarks for the error PSD treatment are given.

2.3 Data properties and preprocessing

The multiple input/output system theory is based on the spectral properties of the data used. This means that the common problems of the discrete Fourier transform are present in the method. Appropriate preprocessing of the observations is needed. Data reductions and terrain corrections must be applied in order to minimize aliasing effects, and the contribution of a global geopotential model must be subtracted for spectral leakage minimization. Both contributions can be restored in the output signal (remove-restore method). In order to minimize the effect of circular convolution, zero-padding must be applied prior to the transformation procedure. Additionally, for an accurate PSD computation, the residual mean value must be subtracted from the input data and restored at the output results. Bendat and Piersol (1986) proposed for an exact PSD computation a trend removal. Nevertheless, a trend must be removed only if it is physically expected or clearly apparent in the data (Bendat and Piersol, 1986).

2.4 Analysis of various PSD estimation procedures

A critical point in the application of this method, as critical as the computation of the covariance function in least-squares collocation, is the estimation of each signal PSD. The methods for PSD estimation can be divided into two basic categories (Marple, 1987): (a) the classical or **non-parametric** methods and (b) the modern or **parametric** methods.

The classical methods are based on FFT computation procedures. According to Marple (1987) and Kay (1987), they can be separated into two techniques:

- The direct non-parametric technique or the so-called **periodogram** approach. In this technique, the PSD is estimated directly from the data using FFT. The PSD of a 2D discrete data field $x[i,j]$ is computed by (Marple, 1987)

$$P_{xx}[k,l] = \lim_{\substack{N\to\infty}}^{M\to\infty} E\left\{ \frac{1}{(2N+1)(2M+1)T_kT_l} \left| T_kT_l \sum_{i=-N}^{N} \sum_{j=-M}^{M} x[i,j] e^{-j2\pi[k_iT_k + l_jT_l]} \right|^2 \right\}, \quad (9)$$

  where M and N are the number of data in each direction, T_k and T_l are the respective record lengths, E is the expectation operator and P_{xx} is the estimated PSD. In common geodetic practice the number of data are limited and the data sample is unique. If the limitation and expectation operators are omitted from the calculation, the PSD can be derived by, e.g., see Bendat and Piersol (1986), Sideris (1996):

$$P_{xx} = X^*X. \quad (10)$$

  The periodogram PSD estimation procedure yields an anisotropic PSD due to the direct involvement of the data set, e.g., Li (1996); Tziavos et al. (1996c). According to previous studies, the use of anisotropic PSD provide better results; see, e.g., Tziavos et al. (1996b); Tziavos et al. (1996a).

- The indirect non-parametric method or the so-called **correlogram** approach. In this technique, the PSD is estimated by the direct transform of the autocorrelation function of the data. If the autocorrelation function C_{xx} is known, the correlogram PSD can be computed following Marple (1987), Sideris (1996), Li (1996):

$$P_{xx}[k,l] = \mathbf{F}\{C_{xx}[m,n]\}. \quad (11)$$

The parametric methods are based on a parametric model description of the data. The parameters of the model are estimated in a least-squares sense. In addition, the degree of expansion of each model is dependent on the data properties and it is computed following certain criteria; e.g., Marple (1987); Cadzow and Ogino (1981); Blais and Vassiliou (1987); Kay (1987). The PSD estimation procedure can be summarized in the following steps:

1. Parametric model selection for the data description
2. Optimal expansion model degree estimation based on adequate criteria
3. PSD computation using the estimated parameters

Two basic models can be developed for 2D data fields. Following Marple (1987), an expression for the Auto-Regressive-Moving-Average (ARMA) model can be given:

$$x[n_1,n_2] = \sum_{k=0}^{q_1}\sum_{m=0}^{q_2} b_{km}\epsilon[n_1-k,n_2-m] - \sum_{k=0}^{p_1}\sum_{m=0}^{p_2} a_{km}x[n_1-k,n_2-m], \quad (12)$$

where $x[n_1,n_2]$ is the 2D data field, $\epsilon[n_1,n_2]$ is a white noise process with variance σ^2, a_{km} are the parameters of the autoregressive part, p_1, p_2 are the expansion degrees of the autoregressive part, b_{km} are the parameters of the moving-average part, and q_1, q_2 are the expansion degrees of the moving-average part. Alternatively, an expression for the Auto-Regressive (AR) model is given by the elimination of b parameters except $b[0,0] = 1$ (Marple, 1987):

$$x[n_1,n_2] = \epsilon[n_1,n_2] - \sum_{k=0}^{p_1}\sum_{m=0}^{p_2} a_{km}x[n_1-k,n_2-m]. \quad (13)$$

The maximum degrees of expansion can be chosen following certain criteria described in Marple (1987); Kay (1987). The coefficients calculation and PSD estimation is performed following a least-squares criterion as presented in Marple (1987) and Cadzow and Ogino (1981). An application of the parametric PSD estimation in 2D data sets can be found in Cadzow and Ogino (1981). The application in 1D data is extensively presented by a number of researchers, e.g., Marple (1987), Kay (1987), Blais and Vassiliou (1987). Numerical examples and applications to gravity field data will be presented in a future paper.

2.5 Noise treatment concept

The main drawback of the multiple input/output method is that the input noise PSD must be known. Only the measurement noise variances are known in practice and not the errors themselves in order to compute directly the error PSD. The basic noise modeling procedures are as follows:

1. *Noise simulation*: A noise field is simulated using a random number generator. Normal as well as uniform distributions can be chosen. The input error PSD is then computed by the direct method using FFT.
2. *PSD models*: Instead of the assumption made for the errors themselves, one can accepts some models for the input noise PSD. White noise models (constant value noise PSD) as well as coloured noise models (non-constant value noise PSD) can be used.

3. *Estimation of input error PSD - Altimetry case*: The repeated tracks of recent satellite altimetry missions provide a PSD estimation procedure. The data samples are more than one due to the Exact Repeat Mission (ERM) configuration. An analysis of PSD estimation from repeated track data is presented in Sailor (1994). In this case, stationarity of the noise can be assumed in practical applications except for the case of "fixed" oceanographic features such as, e.g., the Gulf Stream (Sailor, 1994).

2.6 Analysis in multiple uncorrelated systems

The multiple input/output system in figure 1 can be decomposed in a number of uncorrelated single input/output systems. Let $X_{q\cdot(q-1)!}$ be the residual input data after the subtraction of all other inputs' contributions. In the sequel, the notation of Bendat and Piersol (1986) and Sideris (1996) is followed. The system of figure 1 is modified to a system in which the input signals are uncorrelated with each other; see figure 2. Finally, the equivalence with q uncorrelated sin-

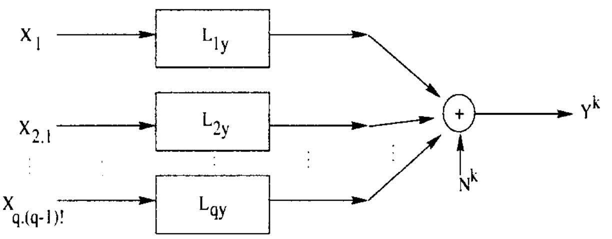

Fig. 2. Multiple uncorrelated input/output systems ($k = 1, \ldots, w$)

gle input/output systems is obvious in figure 3. A recursive

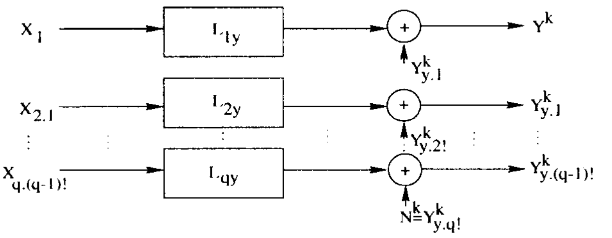

Fig. 3. Individual uncorrelated input/output systems ($k = 1, \ldots, w$)

algorithm is used for the efficient data decomposition. The system of figure 2 is written analytically as:

$$Y = \sum_{i=1}^{q} L_{iy} X_{i\cdot(i-1)!} + N. \tag{14}$$

Let Y represent the $X_{(q+1)}$ measurement. In this case, the output noise represents the $(q+1)$ measurement after the subtraction of the q residual signal contributions. Equation (14) is modified as follows:

$$X_{(q+1)} = \sum_{i=1}^{q} L_{i(q+1)} X_{i\cdot(i-1)!} + X_{(q+1)\cdot q!}. \tag{15}$$

For every $r \leq q$ input signal and $(q+1) = j$, the following equation holds:

$$X_j = \sum_{i=1}^{r} L_{ij} X_{i\cdot(i-1)!} + X_{j\cdot r!}. \tag{16}$$

If the above equation is written for the $(r-1)$ measurement, a recursive formula for the residual signal computation can be derived (Bendat and Piersol, 1986):

$$X_{j\cdot r!} = X_{j\cdot(r-1)!} - L_{rj} X_{r\cdot(r-1)!}. \tag{17}$$

A similar expression can be written for the residual PSD estimation (Bendat and Piersol, 1986):

$$P_{ij\cdot r!} = P_{ij\cdot(r-1)!} - L_{rj} P_{ir\cdot(r-1)!}. \tag{18}$$

For the efficient evaluation of the recursive formulas, the residual transfer function can be computed by (Bendat and Piersol, 1986)

$$L_{iy} = \frac{P_{iy\cdot(i-1)!}}{P_{ii\cdot(i-1)!}} \qquad i = 1, 2, \ldots, q. \tag{19}$$

The recursive algorithm requires less computational effort in comparison with the combined technique. In each run, the user deals with a single input/output system. The equivalence with stepwise least-squares collocation is described in Sideris (1996). The main drawback of the method is the need of a data classification before the computational procedure. More discussion on this subject can be found in Bendat and Piersol (1986), but only for 1D time series. More on the computation procedure and the algorithm can be found in Bendat and Piersol (1986) and Sideris (1996) and some applications in airborne gravimetry are presented by Vassiliou (1986) and Wu and Sideris (1995).

3 Gravity field applications

In this section, some cases will be presented for gravity field applications. Numerical examples will be tabulated for each case.

3.1 *Case 1: Noise filtering in gravity and geoid input data*

The multiple system is presented in figure 4. The input and

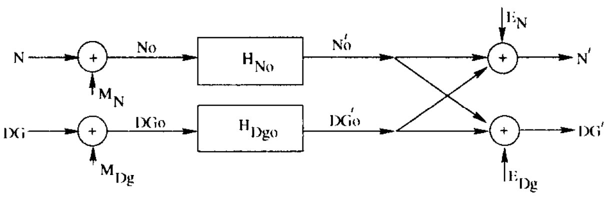

Fig. 4. Filtering of gravity and geoid input signals

the output vector is given by

$$\mathbf{X_o} = \begin{bmatrix} N + M_N \\ \Delta G + M_{\Delta g} \end{bmatrix} \qquad \mathbf{Y_o} = \begin{bmatrix} N' - E_N \\ \Delta G' - E_{\Delta g} \end{bmatrix} \tag{20}$$

In this case, noisy data are imported into the multiple input/output system and the optimal transfer functions are computed in order to minimize the output noise. The results in the output are the filtered data. Assumptions are made related to the input noise PSD, as mentioned in a previous section. The final observation equation is given in matrix notation:

$$\begin{bmatrix} N' \\ \Delta G' \end{bmatrix} = \begin{bmatrix} H_{NN'} & H_{N\Delta G'} \\ H_{\Delta G N} & H_{\Delta G \Delta G'} \end{bmatrix}^T \begin{bmatrix} N + M_N \\ \Delta G + M_{\Delta g} \end{bmatrix} \tag{21}$$

The frequency response matrix is computed by applying (6). The PSD between input and output signal, due to the fact that input and output are the same signals, is given by

$$\mathbf{P_{xy}} = \begin{bmatrix} P_{NN'} & P_{N\Delta G'} \\ P_{\Delta G N} & P_{\Delta G \Delta G'} \end{bmatrix} = \begin{bmatrix} P_{NN} & P_{N\Delta G} \\ P_{\Delta G N} & P_{\Delta G \Delta G} \end{bmatrix} \tag{22}$$

The optimal transfer function matrix is obtained using (6).

3.2 *Case 2: Sea Surface Topography Estimation*

In this case, corrected altimetric Sea Surface Height (SSH) grids from different satellite missions – ERS1 (E1) and Topex / Poseidon (T/P) – are optimally combined with marine gravity anomalies. The output result is a combined estimation of the stationary Sea Surface Topography (SST) signal. The

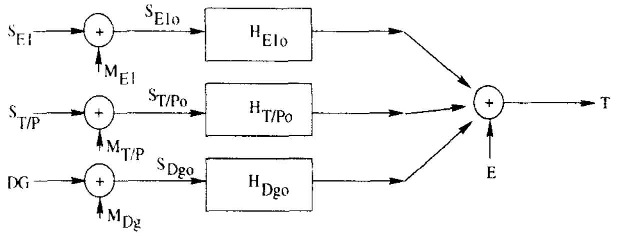

Fig. 5. Sea Surface Topography Estimation

input and output signal vectors are

$$\mathbf{X_o} = \begin{bmatrix} S_{E1} + M_{E1} \\ S_{T/P} + M_{T/P} \\ \Delta G + M_{\Delta g} \end{bmatrix} \qquad \mathbf{Y_o} = \begin{bmatrix} T - E_T \end{bmatrix} \tag{23}$$

The solution equation is given by

$$\begin{bmatrix} T \end{bmatrix} = \begin{bmatrix} H_{S_{E1}T} \\ H_{S_{T/P}T} \\ H_{\Delta GT} \end{bmatrix}^T \begin{bmatrix} S_{E1} + M_{E1} \\ S_{T/P} + M_{T/P} \\ \Delta G + M_{\Delta g} \end{bmatrix} \tag{24}$$

The optimal transfer function matrix is obtained by (6) as

$$\begin{bmatrix} H_{E1T} \\ H_{T/PT} \\ H_{\Delta GT} \end{bmatrix} = \begin{bmatrix} P_{E1oE1o} & P_{E1oT/Po} & P_{E1o\Delta Go} \\ P_{T/PoE1o} & P_{T/PoT/Po} & P_{T/Po\Delta Go} \\ P_{\Delta GoE1o} & P_{\Delta GoT/Po} & P_{\Delta Go\Delta Go} \end{bmatrix}^{-1} \begin{bmatrix} P_{E1T} \\ P_{T/PT} \\ P_{\Delta GT} \end{bmatrix} \tag{25}$$

The cross-PSD between each input and the output signal can be computed using the following procedure:

$$\begin{aligned} P_{E1T} &= S^*_{E1}T = S^*_{E1}(S_{mean} - N) \\ &= S^*_{E1}S_{mean} - S^*_{E1}N \Rightarrow \\ P_{E1T} &= S^*_{E1}\left(\frac{S_{E1} + S_{T/P}}{2}\right) - S^*_{E1}\frac{1}{2\pi\gamma}L_N \Delta G \Rightarrow \\ P_{E1T} &= \frac{S^*_{E1}S_{E1} + S^*_{E1}S_{T/P}}{2} - \frac{1}{2\pi\gamma}L_N S^*_{E1}\Delta G \Rightarrow \\ P_{E1T} &= \frac{P_{E1E1} + P_{E1T/P}}{2} - \frac{1}{2\pi\gamma}L_N P_{E1\Delta g}. \end{aligned} \tag{26}$$

In the above equations, the unknown geoid signal is substituted using Stokes's formula in planar approximation. The planar kernel can be computed directly in the frequency domain with the analytical expression

$$L_N = \frac{1}{\sqrt{u^2 + v^2}} = \frac{1}{q}, \tag{27}$$

or, preferably, can be evaluated through the Fourier transform of the kernel in the space domain

$$L_N = \mathbf{F}\{l_N\} = \mathbf{F}\left\{\frac{1}{\sqrt{x^2 + y^2}}\right\} \tag{28}$$

The cross-PSD between the other input signals and the SST signal are estimated, following similar procedures:

$$P_{T/PT} = \frac{P_{T/PE1} + P_{T/PT/P}}{2} - \frac{1}{2\pi\gamma}L_N P_{T/P\Delta g} \tag{29}$$

$$P_{\Delta gT} = \frac{P_{\Delta gE1} + P_{\Delta gT/P}}{2} - \frac{1}{2\pi\gamma}L_N P_{\Delta g\Delta g} \tag{30}$$

In the abovementioned equations all related quantities are computed from the input signals. The final solution is determined using equation (2).

4 Numerical results

A test area located in the Labrador Sea was selected for some numerical experiments based on real altimetric and marine gravity data. The bounds of the test area are $45.05^o \leq \phi \leq 55.00^o$ and $-55.00 \leq \lambda \leq -45.10$ and the grid intervals are $d\phi = 3'$ and $d\lambda = 6'$. The dimensions of each field are 200 x 100. The statistics of the data fields after the subtraction of a reference geopotential model –EGM96 (Lemoine et al., 1996)– are presented in table 1. The gravity anomaly

Table 1. The statistics of the input signal files

Signal type	max	min	mean	rms	sd
Gravity (mGal)	88.115	-54.968	-2.177	11.174	±10.960
Geoid (m)	0.492	-2.058	-0.937	0.990	±0.320
ERS1 SSH (m)	1.267	-0.623	-0.005	0.253	±0.253
T/P SSH (m)	0.077	-1.131	-0.623	0.640	±0.145

field in the test area is drawn in figure 6. Geoid signal was computed from gravity anomaly signal using the discrete planar FFT technique to evaluate Stokes' convolution integral. Simulated noise values were generated using a gaussian random number generator subroutine with specific standard deviation. The input noise statistics are presented in table 2.

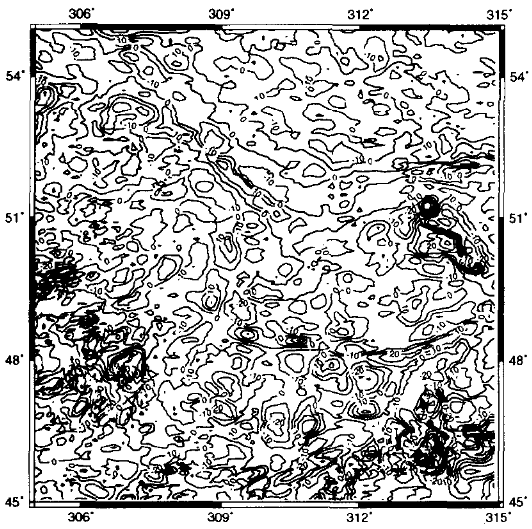

Fig. 6. Gravity signal in the test area (Contour interval 10 mGal)

4.1 *Case 1: Noise filtering in gravity and geoid signal*

Both gravity and geoid signals were introduced into the multiple input/output system. The simulation studies were performed using different input noise levels in order to investigate the noise sensitivity of the system. The differences between the input signal and the output filtered signal are presented in table 3. The output error in gravity field recovery is plotted in figure 7. The results of table 3 show the signal recovery using noisy data. The input noise affects the output signals. Nevertheless, the output noise is 2-3 times less in magnitude than the input noise in gravity signals and 5-10 times less in the geoid signals. This means that most of the input noise is filtered out and only a small portion of it is propagated into the output results. This fact shows that multiple input/output method sensitivity in input noise is quite small. It is worth mentioning that both signals can be recovered totally if only one input noise is applied.

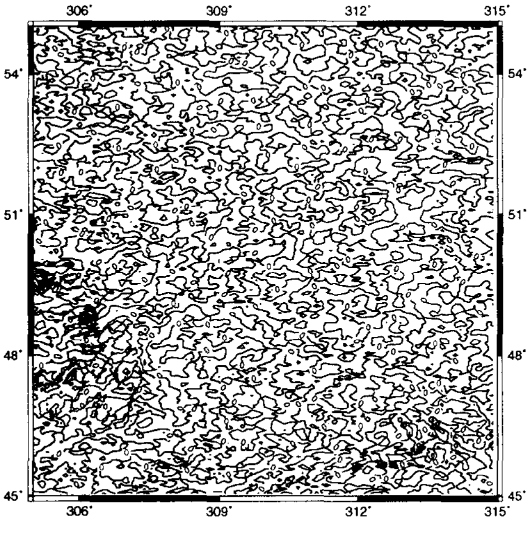

Fig. 7. Prediction error in gravity signal (Contour interval 5 mGal)

Table 2. The statistics of the input noise

Noise type	max	min	mean	rms	sd
Gravity 1 (mGal)	10.742	-14.556	0.023	3.009	±3.009
Gravity 2 (mGal)	22.144	-21.963	-0.026	5.023	±5.023
Geoid 1 (m)	0.208	-0.202	0.000	0.05	±0.05
Geoid 2 (m)	0.387	-0.405	0.000	0.10	±0.10

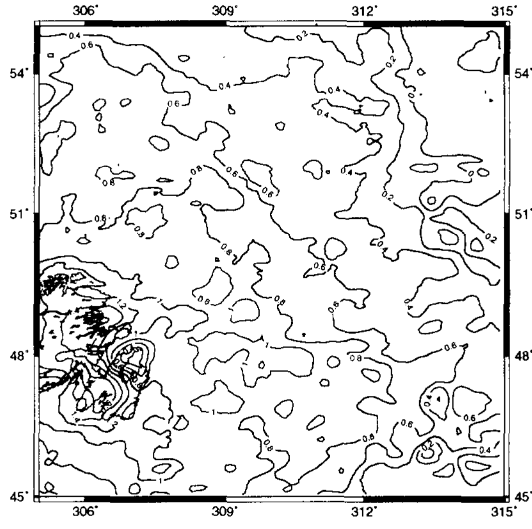

Fig. 8. Estimated SST by multiple i/o system (Contour interval 20 cm)

4.2 *Case 2: SST estimation*

In this case, SSH signals from different satellite missions (ERS1-GM and T/P) were introduced into the multiple input/output system in combination with marine gravity data. The signal statistics are presented in table 1. The input noise level was selected arbitrarily. Nevertheless, altimetric noise was chosen in a manner proportional to each satellite orbit error; see, e.g., LeTraon et al. (1994); LeTraon and Ogor (1998); CERSAT (1994). The gravity noise was chosen with a 5 mGal standard deviation, the ERS1 SSH noise with a 0.10 m and the T/P SSH noise with a 0.05 m standard deviation. A study on the noise level of different satellite missions and error PSD estimation from repeated tracks analysis will be the subject of a future paper. The output SST signal ($SST_{I/O}$) was compared with SST estimated by an average of the two missions (SST_{mean}) where a geoid signal was first computed from the marine gravity data and then subtracted from the SSH data. The statistics of the estimated SST signals are presented in table 4. The estimated SST sig-

Table 3. Differences between input and output signals

Signal type	max	min	mean	rms	sd
Geoid noise sd: **0.05 m**, Gravity noise sd: **3 mGal**					
Geoid (m)	0.037	-0.069	0.000	0.009	±0.009
Gravity (mGal)	8.010	-8.102	0.000	1.305	±1.305
Geoid noise sd: **0.05 m**, Gravity noise sd: **5 mGal**					
Geoid (m)	0.076	-0.052	0.000	0.011	±0.011
Gravity (mGal)	10.258	-9.339	-0.001	1.597	±1.597
Geoid noise sd: **0.10 m**, Gravity noise sd: **3 mGal**					
Geoid (m)	0.067	-0.049	0.000	0.012	±0.012
Gravity (mGal)	8.320	-8.603	0.002	1.434	±1.434
Geoid noise sd: **0.10 m**, Gravity noise sd: **5 mGal**					
Geoid (m)	0.079	-0.064	0.000	0.015	±0.015
Gravity (mGal)	11.470	-10.249	0.002	1.804	±1.804

nal is plotted in figure 8 and the differences between the two methods are presented in figure 9. Comparisons with SST estimated by oceanographic procedures will be carried out in a future study.

Table 4. The statistics of estimated SST signals

	max	min	mean	rms	sd
SST_{mean} (m)	1.804	-0.287	0.623	0.718	±0.357
$SST_{I/O}$ (m)	1.808	-0.267	0.626	0.721	±0.357
Differences (m)	0.046	-0.060	-0.003	0.009	±0.009

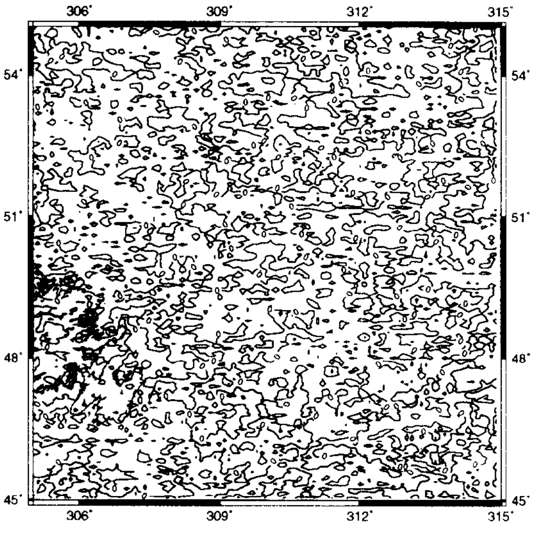

Fig. 9. Differences between estimated SST by multiple input/output method and computed SST from average (Contour interval 2 cm)

5 Summary-Conclusions

Multiple input/output system theory, presented in this paper, is used for efficient heterogeneous data combination. The advantages and drawbacks of the method are listed. Some remarks on the data preprocessing and data properties are given and an analysis of various PSD estimation procedures is presented. The decomposition of the multiple input/output system into multiple single uncorrelated systems is outlined. In addition, some gravity field applications are presented and numerical tests are carried out. Data sets from marine gravity anomaly measurements, geoid signal and SSH signal from two different altimetric missions are optimally combined in order to filter out the noise and estimate the stationary part of SST. Simulated input noises are introduced to the signals in order to avoid the singularities in frequency response function matrix computation; see, e.g., Bendat and Piersol (1980), Bendat and Piersol (1986), Sansò and Sideris (1997).

In this spectral method, the input errors can be propagated into the system output results. The effect of the input noise is studied. It is shown that the estimation and use of an optimal frequency response function can filter-out most of the input noise. The sensitivity of the method to the input noise is small. Some assumptions on the input noise PSD are needed for the efficient estimation of cross-PSD between input and output signals. These assumptions are introduced through simulation noise, PSD models or direct computation in repeated tracks altimetry data.

The method's computer storage requirements are minimized due to the limited matrix dimensions. A comparison between matrix dimensions in input/output method and least-squares collocation is presented in Sideris (1996). Nevertheless, the non-stationarity nature of the noise complicates the spectral solution: The simple algebraic equations are replaced by integral equations which makes this solution as inefficient to compute as in the space domain.

The altimetric applications of multiple input/output system theory seem very promising for the combination of various satellite missions and the estimation of noise models for these data. In addition, the application of parametric models in PSD estimation using irregulary distributed data will provide the possibility of using the original measurements, thus eliminating the gridding error.

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