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Effect of intrinsic-gain fluctuations on quantum noise of phosphor materials used in medical X-ray imaging

N. Kalivas[1], L. Costaridou[1], I. Kandarakis[2], D. Cavouras[2], C.D. Nomicos[3], G. Panayiotakis[1,*]

[1] Department of Medical Physics, School of Medicine, University of Patras, 265 00 Patras, Greece
[2] Department of Medical Instrumentation Technology, Technological Educational Institution of Athens, Ag. Spyridonos Street, Aigaleo, Athens, Greece
[3] Department of Electronics, Technological Educational Institution of Athens, Ag. Spyridonos Street, Aigaleo, Athens, Greece

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Abstract. The quality of a medical image depends, among other parameters, on quantum noise. Quantum noise is affected by the fluctuations in the number of optical quanta produced within the phosphor, per absorbed X-ray (i.e. phosphor intrinsic-gain fluctuations). This effect is considered by means of a factor, called in this study intrinsic-gain noise factor, IGNF(E). In existing theoretical models of quantum noise, the corresponding factor is taken to be equal to one. In this paper, an expression that accounts for the coefficient of variation of the phosphor intrinsic gain is introduced. This expression takes into account the process of electron–hole pair conversion to optical photons and the frequency distribution function of the emitted optical photon energy. Subsequently IGNF(E) is expressed in terms of this coefficient of variation. IGNF(E) has been calculated for several phosphors and for various energies. For all medical X-ray energies studied, phosphors that exhibit a high relative fluctuation of emitted optical photon energy, IGNF(E) exceeds by 2% to over 17% the corresponding factor of the existing theoretical models of quantum noise.

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Phosphors doped with various activators (Eu, Tb, Ag, etc.) are widely used in medical imaging as X-ray-to-light converters in conjunction with optical detectors (films, photodiodes, etc.) [1–3]. The quality of a medical image depends, among other parameters, on quantum noise. Quantum noise is affected by the number of optical quanta, produced within the phosphor, per absorbed X-ray of energy E (i.e. phosphor intrinsic gain) [4, 5]. Phosphor intrinsic gain is subject to statistical fluctuations. Thus for equal X-ray energy absorption, unequal number of optical quanta are produced [4–7]. Quantum noise is proportional to the variation of the intrinsic phosphor gain, which reflects the statistical fluctuations in the production of optical quanta produced by the activator material within the phosphor. Experimental methods have been reported to determine the fluctuation of the optical quanta emitted by the phosphor per absorbed X-ray [8–10]. However, to the best of our knowledge, no experimental method has been introduced, to measure the fluctuation of intrinsic phosphor gain. In addition, the contribution to quantum noise, called in this study intrinsic-gain noise factor (IGNF(E)), in existing theoretical models, of the fluctuation of phosphor intrinsic gain, is unity, as the gain procedure is assumed to follow Poisson distribution [2, 4–6, 11].

In this paper, an alternative expression of phosphor intrinsic-gain fluctuation is presented, which can contribute to quantum-noise modeling. This expression takes into account the process of electron–hole pair conversion to optical photons and the frequency distribution function of the emitted optical photon energy. IGNF(E) incorporating the above expression has been calculated for several phosphor materials and for various energies, based on X-ray conversion efficiency and optical emission spectral data.

1 Method and materials

Quantum noise has been found to be proportional to a factor accounting for the fluctuation in the number of optical quanta produced per absorbed X-ray within the phosphor. This factor which will be called hereafter intrinsic-gain noise factor, IGNF(E), equals [2, 4–6, 11]:

$$\mathrm{IGNF(E)} = \left[\frac{S^2(m_\mathrm{o}(E))}{\overline{m_\mathrm{o}}^2(E)} - \frac{1}{m_\mathrm{o}(E)} + 1 \right] , \tag{1}$$

where $\overline{m_\mathrm{o}}(E)$ is the mean phosphor intrinsic gain and $\mathrm{SD}[m_o(E)]$ is its standard deviation [5, 6]. When the gain procedure follows Poisson distribution, $S^2[m_\mathrm{o}(E)] = \overline{m_\mathrm{o}}(E)$ and therefore $\mathrm{IGNF(E)} = 1$.

The intrinsic gain of the phosphor material has been previously reported to be equal to [2, 6]:

$$m_\mathrm{o}(E) = \frac{E}{E_\lambda} n_\mathrm{C} , \tag{2}$$

* Corresponding author.
(Fax: +30-61/992-496, E-mail: panayiot@upatras.gr)

where E is the absorbed X-ray energy, E_λ is the energy of the optical photons and n_C is the intrinsic conversion efficiency of the phosphor expressing the fraction of the absorbed X-ray energy converted to light [12–14].

When an X-ray quantum interacts with the phosphor and deposits its energy E, within the lattice of the phosphor, electron–hole pairs are created. The average energy required to create an electron–hole pair of energy E_g (fundamental electronic band gap of the host material) equals βE_g. β is a constant that characterize the excess energy above E_g that should be absorbed, in order for an electron–hole pair to be created [12, 14–16]. Therefore the total number of electron–holes created equals $E/\beta E_g$. The theoretical value of β is 1.5 [15], but in several cases its value has been experimentally determined to be higher [12, 14, 16].

A fraction, R, of the created electron–holes, will reach the luminescent center and subsequently a fraction q will be absorbed in the activator [12, 14, 17]. From the absorbed electron–hole pairs only a fraction β' will actually contribute to optical photon creation. Therefore the number of the electron–holes that actually contribute to optical photon emission, X, is

$$X = \frac{E}{\beta E_g} q R \beta' \,. \tag{3}$$

The product $qR\beta'$ is the fraction of the generated electron–holes that actually yield optical photons.

The energy carried by these electron–hole pairs equals XE_g. If E_λ is the optical photon energy, then the number of optical photons that will be produced is:

$$N_\lambda(E) = \frac{XE_g}{E_\lambda} \,. \tag{4}$$

Phosphor intrinsic gain, $m_o(E)$, is defined as the number of optical photons produced per absorbed X-ray of energy E [1, 2, 11].

By taking into account the above definition of phosphor intrinsic gain and (4), $m_o(E)$ can also be expressed as:

$$m_o(E) = N_\lambda(E) = \frac{X}{E_\lambda} E_g \,. \tag{5}$$

The process of yielding optical photons from electron–hole pairs in the activator can be considered random. Furthermore the energy of an optical photon is not predetermined, as the activator could have one or more energy levels and each level is characterized by a random broadening mechanism of the corresponding emission line [17, 18]. Therefore $m_o(E)$, E_λ, and X in (2) and (5) could be treated by means of their corresponding expected values, $\overline{m_o}(E)$, $\overline{E}_\lambda$, and $\overline{X}$ respectively. As a consequence each one of $m_o(E)$, X, and E_λ randomly fluctuates about its mean value.

An expression of the fluctuations of the gain can be given by means of its mean-square error [16, 19, 20]. However for such an expression to be valid the following conditions should be met: (i) X and E_λ should be uncorrelated, (ii) X and E_λ are randomly distributed and their corresponding variances $S(X)$ and $S(E_\lambda)$ are known. Condition (i) is valid since the number of electron–hole pairs absorbed and inducing optical photons is a function of absorbed energy E, while the energy distribution of the optical photons is a function of the activator [16–18]. Condition (ii) is valid since the process of an electron–hole pair to yield optical photons can be assumed to follow a binomial distribution. This is so since one electron–hole pair may either yield optical photons or not, and each event can be considered independent.

For the case of optical photon energy, data describing the frequency distribution function, $P(E_\lambda)$, of the optical photons per wavelength were obtained. The emission spectrum was measured with an Oriel 7240 grating monochromator. These data were corrected for the optical response of the monochromator and the background, in order to diminish any systematic errors in E_λ calculations.

The expected mean-square error in the gain of the phosphor equals [16, 19, 20]:

$$S^2(m_o(E)) = \left(\frac{\partial(m_o(E))}{\partial E_\lambda} S(E_\lambda)\right)^2 + \left(\frac{\partial(m_o(E))}{\partial X} S(X)\right)^2 \,, \tag{6}$$

where ∂ stands for the partial derivative.

Subsequently:

$$S^2(m_o(E)) = \frac{X^2 E_g^2}{\overline{E}_\lambda^4} S^2(E_\lambda) + \frac{E_g^2 S^2(X)}{\overline{E}_\lambda^2} \,, \tag{7}$$

where the partial derivatives are evaluated at the mean optical photon energy $\overline{E}_\lambda$ and the mean number of electron–hole pairs $\overline{X}$ [16, 20].

Since the frequency distribution function of E_λ, $P(E_\lambda)$, is available then the fluctuations of the optical photon energy with respect to the mean value can be derived straightforwardly as [16, 19, 20]:

$$S^2(E_\lambda) = \left(\int_{E_{\lambda 1}}^{E_{\lambda 2}} (E_\lambda - \overline{E}_\lambda)^2 P(E_\lambda)\, \mathrm{d}E_\lambda\right) \,, \tag{8}$$

where $E_{\lambda 1}$ and $E_{\lambda 2}$ are the minimum and maximum energies of the optical photons.

Meanwhile, $\overline{E}_\lambda$ equals to:

$$\overline{E}_\lambda = \left(\int_{E_{\lambda 2}}^{E_{\lambda 1}} E_\lambda P(E_\lambda)\, \mathrm{d}E_\lambda\right) \,. \tag{9}$$

In medical X-ray imaging energies, the number of electron–holes generated in the phosphor, $E/\beta E_g$, is large. Furthermore as mentioned above, since the process of yielding optical photons from electron–hole pairs is assumed to follow a binomial distribution, the term $qR\beta'$ can be assumed as the probability of an electron–hole to yield optical photons. Furthermore by comparing (2) and (5) we obtain $qR\beta' = n_C\beta$. Therefore for the binomial distribution, the fluctuation in the number of electron–hole pairs equals to [16, 19, 20]:

$$S^2(X) = \frac{E}{\beta E_g} n_C \beta (1 - n_C \beta) \,. \tag{10}$$

Hence due to (5), (6), (7), and (10):

$$S(m_o(E)) = \frac{En_C\beta}{\beta\overline{E}_\lambda}\sqrt{\frac{S^2(E_\lambda)}{\overline{E}_\lambda^2} + \frac{\beta E_g}{E}\frac{1-n_C\beta}{n_C\beta}}\,. \quad (11)$$

Equation (11) expresses the gain fluctuations of an activator in the phosphor when energy E is absorbed. Furthermore, due to (2), $(En_C\beta)/(\beta\overline{E}_\lambda)$ is the mean value of the gain. Therefore, the ratio of the standard error to the mean value is simply derived by the following formula:

$$\frac{S(m_o(E))}{\overline{m}_o(E)} = \sqrt{\frac{S^2(E_\lambda)}{\overline{E}_\lambda^2} + \frac{E_g}{E}\frac{1-n_C\beta}{n_C}}\,. \quad (12)$$

This ratio takes into account the fluctuations of the gain with respect to its mean value (coefficient of variation). The first factor in the sum under the square root accounts for the square of the relative fluctuation of the emitted optical photon energy. The second factor accounts for the square of the relative fluctuation of the number of the electron–hole pairs contributing to light emission in the luminescent center, for an absorbed X-ray photon of energy E.

By substituting (12) in (1) it is obtained that:

$$\mathrm{IGNF(E)} = \left[\frac{S^2(E_\lambda)}{\overline{E}_\lambda^2} + \frac{E_g}{E}\frac{1-n_C\beta}{n_C} - \frac{1}{\overline{m}_o(E)} + 1\right]. \quad (13)$$

Following, IGNF(E) was determined for various phosphor materials. Phosphor samples were supplied in powder form by Derby Luminescent Ltd. (Code No.1510) and by Riedel de Haen-Lumilux Ltd. (Code No. 54009). The frequency distribution of E_λ, $P(E_\lambda)$, of the phosphor was obtained as previously described. The intrinsic conversion efficiency, n_C, was taken from literature [17, 21–26].

2 Results and discussion

In Fig. 1 the normalized spectra, $P(E_\lambda)$, of various phosphors are presented. It can be seen that YVO_4:Eu^{3+} and Y_2O_3:Eu^{3+} emit a narrow line around 620 nm. On the other hand CsI:Na has a broad emission spectrum ranging from 300 nm to 620 nm. The other phosphors emit with several peaks. The relative probability of emission of their secondary peaks, with respect to their major peak is either high (for example Y_2O_2S:Tb), or low (for example La_2O_2S:Tb).

Table 1. The intrinsic conversion efficiency (n_C), energy gap (E_g), relative fluctuation of the emitted optical photon energy, $S(E_\lambda)/\overline{E}_\lambda$ and the mean optical photon energy ($\overline{E}_\lambda$), of the phosphors used in this study

Phosphor	Efficiency (n_C)	E_g / eV	$S(E_\lambda)/\overline{E}_\lambda$	$\overline{E}_\lambda$ / eV
Y_2O_3:Eu^{3+}	0.095	5.6	0.045	2.02
YVO_4:Eu^{3+}	0.070	8.0	0.020	2.00
La_2O_2S:Tb	0.180	4.4	0.056	2.26
Y_2O_2S:Tb	0.180	4.6	0.154	2.66
Gd_2O_2S:Tb	0.190	4.5	0.142	2.46
CsI:Na	0.110	6.4	0.416	2.94

In Table 1 the intrinsic conversion efficiency, n_C, the energy gap energy, E_g, as well as the relative fluctuation of the emitted optical photon energy, $(S(E_\lambda))/\overline{E}_\lambda$, and the mean optical photon energy $\overline{E}_\lambda$ are listed. $(S(E_\lambda))/\overline{E}_\lambda$ and $\overline{E}_\lambda$ were calculated by using (8) and (9). $E_{\lambda 1}$ and $E_{\lambda 2}$ were determined from the corresponding spectrum (Fig. 1). The values for n_C and E_g were obtained from the literature [3, 12].

In Table 2, $(S(m_o(E)))/(\overline{m}_o(E))$ values are presented for various energies E (5, 15, 45, and 60 keV). The values on the left in each cell of the table correspond to $\beta = 1.5$ [15], whereas the values on the right to $\beta = 2.8$, which is experimentally determined [12].

It can be seen that $(S(m_o(E)))/(\overline{m}_o(E))$ varies with energy, for energies up to 15 keV for all phosphors. Above 15 keV the corresponding changes are minimal for Y_2O_2S:Tb, Gd_2O_2S:Tb and CsI:Na phosphors. This is expected, since (12) is affected by the relative fluctuation in the emitted light photon energy, as well as by the relative fluctuation in the number of electron–hole pairs absorbed in the lumi-

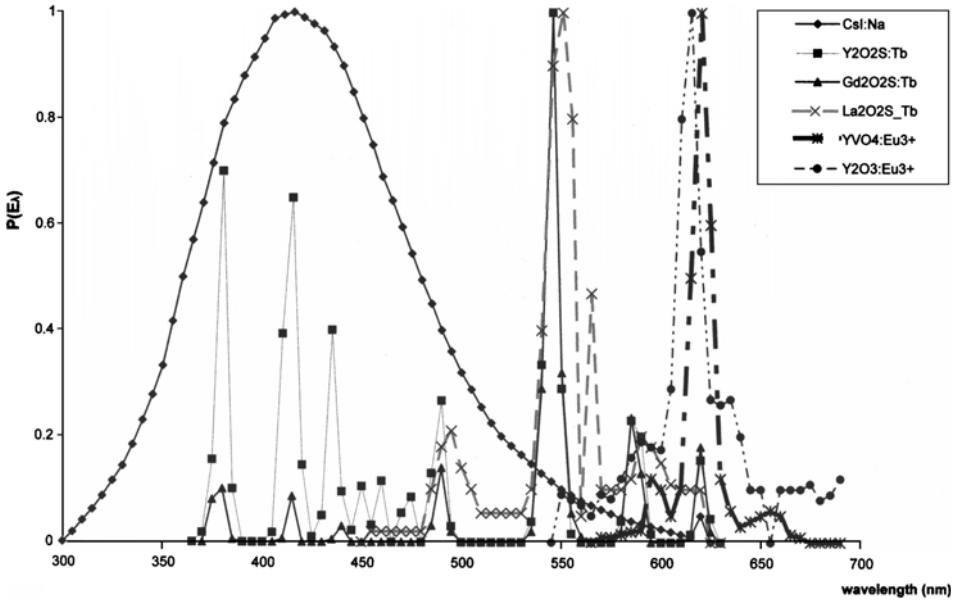

Fig. 1. The frequency distribution function of the emitted optical photon energy for CsI:Na, Y_2O_2S:Tb, Cd_2O_2S:Tb, La_2O_2S:Tb, YVO_4:Eu^{3+}, and Y_2O_3:Eu^{3+} phosphors

Table 2. The coefficient of variation of phosphor intrinsic gain, $(S(m_o(E)))/\overline{m}_o(E)$, for various phosphors and energies. Values to the left correspond to $\beta = 1.5$, whereas values to the right correspond to $\beta = 2.8$

Phosphor Energy / keV	Y_2O_3:Eu^{3+}	YVO_4:Eu^{3+}	La_2O_2S:Tb	Y_2O_2S:Tb	Gd_2O_2S:Tb	CsI:Na
5	0.110, 0.103	0.144, 0.137	0.085, 0.078	0.167, 0.163	0.153, 0.150	0.431, 0.427
15	0.073, 0.070	0.085, 0.081	0.069, 0.066	0.159, 0.158	0.146, 0.145	0.424, 0.423
45	0.056, 0.054	0.052, 0.049	0.063, 0.062	0.156, 0.156	0.143, 0.143	0.422, 0.422
60	0.053, 0.052	0.046, 0.044	0.062, 0.062	0.156, 0.156	0.143, 0.143	0.422, 0.422

nescent center and yield light photons. For low energies, fewer electron–hole pairs are created, characterized by increased relative fluctuation of X. $(S(m_o(E)))/(\overline{m}_o(E))$ is more affected by these fluctuations than the corresponding fluctuations on the optical photon energy, especially for phosphors characterized by a narrow spectrum.

In the case of absorbed X-ray energies above 15 keV, the number of electron–hole pairs contributing to light emission per absorbed X-ray is large. This accounts for decreased relative fluctuations in X. Therefore the relative contribution of the fluctuations in the optical photon energy, E_λ, in (12) is relatively increased, as the fluctuation in the number of electron–hole pairs is decreased with increasing X-ray energy.

It can be seen from Tables 1 and 2, that phosphors that are characterized by a broad spectrum, $P(E_\lambda)$ and high $(S(E_\lambda))/\overline{E}_\lambda$ value (i.e. CsI:Na), present high $(S(m_o(E)))/(\overline{m}_o(E))$ in all energies. Furthermore phosphor materials with optical spectra containing three or four main, with high relative frequency, characteristic peaks (for example Y_2O_2S:Tb and Gd_2O_2S:Tb) also exhibit high $(S(E_\lambda))/\overline{E}_\lambda$ values and subsequently $(S(m_o(E)))/(\overline{m}_o(E))$ high for all energies. The two Eu^{3+} activated phosphors studied in this work, YVO_4:Eu^{3+} and Y_2O_3:Eu^{3+}, are characterized by narrow-band optical spectra and may well be considered "monochromatic". As they are characterized by lower $(S(E_\lambda))/\overline{E}_\lambda$ than other phosphors (see Table 1), they exhibit lower $(S(m_o(E)))/(\overline{m}_o(E))$ values for all energies. However for low energies these phosphors are more affected by the relative fluctuations of X, which can explain the higher value of $(S(m_o(E)))/(\overline{m}_o(E))$ as compared to La_2O_2S:Tb.

In Table 3, IGNF(E) values are presented for various energies E (5, 15, 45, and 60 keV). The values on the left in each cell of the table correspond to $\beta = 1.5$ [15], whereas the values the right correspond to $\beta = 2.8$, which is experimentally determined [12].

IGNF(E) values for the narrow-band-emitting phosphors (Y_2O_3:Eu^{3+} and YVO_4:Eu^{3+}), are very close to unity, for all the energies considered, due to their low $(S(E_\lambda))/\overline{E}_\lambda$ values. The same holds for La_2O_2S:Tb, which is also characterized by low $(S(E_\lambda))/\overline{E}_\lambda$. Gd_2O_2S:Tb and Y_2O_2S:Tb, which both have higher $(S(E_\lambda))/\overline{E}_\lambda$ values, are characterized by higher IGNF(E), ranging from 1.025 to 1.020. CsI:Na, which has a broad frequency distribution function and a higher $(S(E_\lambda))/\overline{E}_\lambda$ value, has an IGNF(E) value ranging from 1.18 to 1.177, which is the highest obtained in this study. The calculated IGNF(E) values for Gd_2O_2S:Tb, Y_2O_2S:Tb and CsI:Na phosphors exceeds by 2% to over 17% the corresponding factors of the existing theoretical models of quantum noise where, IGNF(E) = 1. The observed differences can be attributed more to the frequency distribution function of the emitted optical photon energy and less to the absorbed X-ray energy.

By comparing Tables 2 and 3 it may be observed that $(S(m_o(E)))/(\overline{m}_o(E))$ values are more dependent on the absorbed X-ray energy, for all phosphors, except for Gd_2O_2S:Tb, Y_2O_2S:Tb, and CsI:Na for high energies. Thus (12) could be used in addition to IGNF(E) to characterize IGNF(E) dependence on absorbed X-ray energy.

3 Conclusion

An alternative expression of the intrinsic-gain fluctuation of phosphor materials excited by medical X-rays, which takes into account the process of electron–hole pair conversion to optical photons and the frequency distribution function of the emitted optical photon energy has been presented. This expression, which is the coefficient of variation of the phosphor intrinsic gain, accounts for the relative fluctuations of the gain with respect to its mean value. Subsequently, the influence of the gain fluctuations on quantum noise, by means of IGNF(E), was expressed in terms of this coefficient of variation and was calculated for several phosphors. For all medical X-ray energies, phosphors having a low value corresponding to their relative fluctuation of the emitted optical photon energy, $(S(E_\lambda))/\overline{E}_\lambda$, were found to have an IGNF(E) approximately equal to 1. On the other hand, in phosphors with high $(S(E_\lambda))/\overline{E}_\lambda$ values, IGNF(E) was found to exceed by 2% to over 17% the corresponding factors of the existing theoretical models of quantum noise, where IGNF(E) = 1.

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Table 3. Intrinsic-gain noise factor, IGNF(E), for various phosphors and energies. Values to the left correspond to $\beta = 1.5$, whereas values to the right correspond to $\beta = 2.8$

Phosphor Energy / keV	Y_2O_3:Eu^{3+}	YVO_4:Eu^{3+}	La_2O_2S:Tb	Gd_2O_2S:Tb	Y_2O_2S:Tb	CsI:Na
5	1.008, 1.006	1.015, 1.013	1.005, 1.004	1.021, 1.020	1.025, 1.024	1.181, 1.179
15	1.004, 1.003	1.005, 1.005	1.004, 1.004	1.020, 1.020	1.024, 1.024	1.178, 1.177
45	1.003, 1.003	1.002, 1.002	1.004, 1.004	1.020, 1.020	1.024, 1.024	1.177, 1.177
60	1.003, 1.003	1.002, 1.002	1.004, 1.004	1.020, 1.020	1.024, 1.024	1.177, 1.177

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