PERISTALTIC PUMPING OF A PARTICULATE SUSPENSION IN A CATHETERIZED TUBE

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Abstract

Peristaltic pumping of a particle-fluid suspension in a catheterized circular tube has been investigated. The coupled differential equations for both the fluid and the particle phases have been solved and the expressions for the flow rate, pressure drop, friction forces at the tube and the catheter wall have been derived. It is found that the pressure drop increases with the flow rate for any given particle concentration, catheter size and amplitude ratio. Also for any given flow rate and the catheter size, pressure drop increases with the particle concentration, and assumes significantly higher magnitude in a catheterized tube than its corresponding value in uncatheterized tube. The friction forces (at tube as well as the catheter wall) possess characteristics similar to the pressure drop (an opposite characteristics to the pressure rise) with respect to any parameter. The friction force at the tube wall is found to be significantly higher in magnitude than the corresponding friction force at the catheter wall.

Keywords: Peristaltic, catheter, flow-rate, pressure drop, friction force, particle concentration.

INTRODUCTION

Since the first investigation of Latham (1966), fluid transport through flexible tubes by means of peristaltic wave motion of the tube wall has been the subject of engineering and scientific research for over four decades. The peristaltic motion consists in a narrowing and traverse shortening of a portion of the tube, which then relaxes while a lower portion becomes shortened and narrowed. During the course of the peristaltic motion the tube wall is excited by traveling waves that cause the points on the tube transversally to the direction of the fluid motion. Peristaltic transport is therefore a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible duct containing liquid or mixture. The phenomenon of such transport is called peristalsis. Heart-lung machine, finger and roller pumps have been fabricated using the mechanism of peristalsis. Some aquatic animals use peristalsis as a means of locomotion. Besides, it's various engineering applications, it is also known to be responsible for fluid transport in many biological organs including in the vasomotion of small blood vessels such as arterioles, venules and capillaries (Srivastava and Srivastava, 1984).

Shapiro et al. (1969) and Jaffrin and Shapiro (1971) explained the basic principles and brought out clearly the significance of the various parameters governing the flow. In various details, a review of much of the early literature up to the year 1983 was presented in an article by Srivastava and Srivastava (1984). The significant contributions between the years 1984 and 1994 are referenced well in Srivastava and Saxena (1995). The important studies of recent years include the investigations of Srivastava and Srivastava (1997), Mekheimer et al. (1998), Muthu et al. (2001), Srivastava (2002), Misra and Pandey (2002), Hayat et al. (2002,2003,2004), Mekheimer(2003), Misra and Rao (2004), Hayat et al. (2005), Hayat and Ali (2006 a, b), Srivastava (2007a), Medhavi and coworkers (2008 a,2008b,2009), Hayat and Coworkers (2008 a,b), Ali and Hayat (2008), and a few others.

The theory of particulate suspension is very useful in understanding of a number of diverse physical problems concerned with powder technology, fluidization, sedimentation, combustion, aerosol filtration, atmospheric fallout, lunar ash flow, environmental pollution, etc. Most recently, the interest has developed in applying the theory of particle-fluid suspension to physiological flows including the vasomotion of small blood vessels such as arterioles, venules and capillaries. Peristaltic pumping of a particle-fluid mixture has been investigated by Hung and Brown (1976), Takabatake and Ayakawa (1982), Srivastava and coworkers (1989, 1997, 2002), Mekheimer et al. (1998), Medhavi and Singh (2008 b, 2009) and several others.

The use of catheters is of immense importance in many areas of technical importance and has become a standard tool for diagnosis and treatment of certain cardiovascular diseases (McDonald, 1986, Back and Coworkers, 1994, 1996; Sarkar and Jayaraman 1998; Sankar and Hemlatha, 2007) in modern medicine. The mathematical model corresponds to the flow in the annular space of two concentric tubes. The geometrically similar biomechanical problem of peristaltic flow to study the effects of inserted catheter on ureteral flow was analyzed by Roos and Lykoudis (1970). A number of authors including Hakeem et al. (2002), Hayat et al. (2006) and most recently Srivastava (2007a) have explained the effects of an endoscope on flow of chyme in gastrointestinal tract.

The aim of the present investigation is to study the peristaltic pumping of a particulate suspension in a circular tube with an inserted catheter. In view of the observations (Srivastava, 1995, 2007b) that the particulate suspension model (i.e., a suspension of red blood cells in plasma) is of particular importance to study the flow of blood in narrow arteries, it is strongly believed that the research reported here may be applied to explain to peristaltic induced flow of blood through narrow catheterized arteries.

FORMULATION OF THE PROBLEM

Consider the axisymmetric flow of a particle-fluid mixture in a circular cylindrical tube of radius a with an inserted catheter of radius a_1. The catheter wall is rigid and the tube wall is flexible upon which are imposed sinusoidal peristaltic waves of finite amplitude traveling down its wall. The geometry of the wall surface of the tube is described (Fig. 1) as
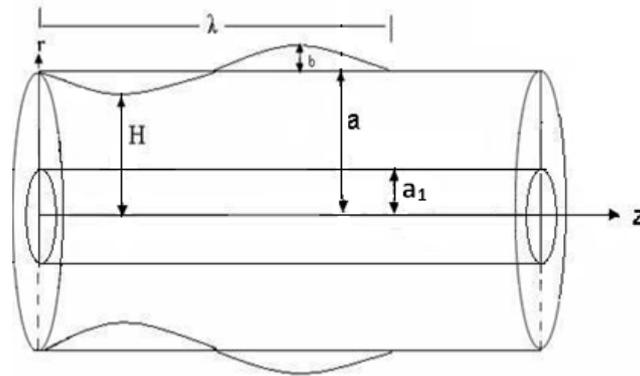
Fig.1 Geomertical representation of peristaltic waves
in a circular tube with an inserted catheter.

$$H(z, t) = a + b \sin \frac{2\pi}{\lambda} (z - ct), \tag{1}$$

where b $(0 \le b \le a - a_1)$ is the amplitude of the wave, λ ($\le$ L, the length of the tube under consideration) is the wavelength, c is the wave propagation speed, t is the time and z is the axial coordinate.

The equations governing the linear momentum and the conservation of mass for both the fluid and particle phases using a continuum approach are expressed (Drew, 1979; Srivastava and Srivastava, 1997) as

$$(1-C) \rho_f \left\{ \frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial z} + v_f \frac{\partial u_f}{\partial r} \right\} = -(1-C) \frac{\partial p}{\partial z} + (1-C) \mu_s (C)$$

$$x \left(\frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial}{\partial r}) + \frac{\partial^2}{\partial z^2} \right) u_f + CS (u_p - u_f), \tag{2}$$

$$(1-C) \rho_f \left\{ \frac{\partial v_f}{\partial t} + u_f \frac{\partial v_f}{\partial z} + v_f \frac{\partial v_f}{\partial r} \right\} = -(1-C) \frac{\partial p}{\partial r} + (1-C) \mu_s (C)$$

$$x \left(\frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial}{\partial r}) - \frac{1}{r^2} + \frac{\partial^2}{\partial z^2} \right) v_f + CS (v_p - v_f), \tag{3}$$

$$\frac{1}{r} \frac{\partial}{\partial r} \left[r (1-C) v_f \right] + \frac{\partial}{\partial z} \left[(1-C) u_f \right] = 0, \tag{4}$$

$$C \rho_p \left\{ \frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z} + v_p \frac{\partial u_p}{\partial r} \right\} = -C \frac{\partial p}{\partial z} + CS (u_f - u_p), \tag{5}$$

$$C \rho_p \left\{ \frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial z} + v_p \frac{\partial v_p}{\partial r} \right\} = -C \frac{\partial p}{\partial r} + CS (v_f - v_p), \tag{6}$$

$$\frac{1}{r} \frac{\partial}{\partial r} (r C v_p) + \frac{\partial}{\partial z} (C u_p) = 0, \tag{7}$$

where r is the radial coordinate measured in the direction normal to the tube axis, (u_f, v_f) denotes the fluid phase and (u_p, v_p) denotes the particle phase velocity components along (z, r) directions, respectively; C be the volume fraction of particulate phase; ρ_f and ρ_p be the actual densities of the material constituting fluid and particulate phases, respectively; $(1-C)\rho_f$ is the fluid phase density, $C\rho_p$ the particulate phase density, p denotes the pressure, $\mu_s(C) \cong \mu_s$ is the mixture viscosity and S being the drag coefficient of interaction for the force exerted by one phase on the other. The concentration of the particles is assumed to be small enough so as to neglect the field interaction among them (Srivastava, 1996). The volume fraction density, C of the particles is chosen to be a constant which is a good approximation for the low concentration of small particles (Batchelor, 1974, 1976).

An empirical relation for the suspension viscosity, μ_s has been chosen for the present problem (Charm and Kurland, 1974; Srivastava and Srivastava, 1989) as

$$\mu_s (C) = \frac{\mu_o}{1 - mC} , \qquad (8)$$

where m=0.070 exp [2.49C+ (1107/T) exp (-1.69C)]; μ_o is the fluid viscosity (suspending medium) and T is the temperature of the mixture measured in absolute scale (K). The viscosity of the suspension expressed by this formula (eqn. (8)) is found to be reasonably accurate up to C=0.6 (i.e., 60% particle concentration; Srivastava and coworkers, 1989, 1996, 1997; Charm and Kurland, 1974).

The expression for the drag coefficient of interaction, S for the study is selected (Tam, 1969) as

$$S = \frac{9}{2} \frac{\mu_o}{a_o^2} \quad \frac{4 + 3[8C - 3C^2]^{1/2} + 3C}{(2 - 3C)^2} , \qquad (9)$$

with a_o as the radius of a particle.

Introducing the following dimensionless variables

r'= r/a , z' =z/λ , (u'_f, u'_p) = (u_f, u_p) / c , (v'_f, v'_p) = λ (v_f, v_p) / ac , t' = ct/λ , p' = a^2p / λ c μ_o, S' = S a^2/μ_o , μ = μ_s/μ_o ,

into equations (2) – (7), after dropping primes, yields the following

$$(1-C) \, \delta \, Re \left\{ \frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial z} + v_f \frac{\partial u_f}{\partial r} \right\} = -(1-C) \frac{\partial p}{\partial z} + (1-C) \, \mu$$

$$x \left\{ \frac{1}{r} \frac{\partial}{\partial r} (r \frac{\partial u_f}{\partial r}) + \delta^2 \frac{\partial^2 u_f}{\partial z^2} \right\} + CS(u_p - u_f), \qquad (10)$$

$$(1\text{-}C)\ \delta\ \mathrm{Re}\left\{\frac{\partial v_f}{\partial t}+u_f\frac{\partial v_f}{\partial z}+v_f\frac{\partial v_f}{\partial r}\right\}=-(1\text{-}C)\frac{\partial p}{\partial r}+(1-C)\ \mu$$

$$\times\ \delta^2\left\{\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial(rv_f)}{\partial r}-\frac{1}{r^2}+\delta^2\frac{\partial^2 v_f}{\partial z^2}\right\}+CS$$

$$\delta^2\,(v_p-v_f),\qquad(11)$$

$$\frac{1}{r}\ \frac{\partial}{\partial r}\Big[r\,(1-C)\,v_f\Big]+\frac{\partial}{\partial z}\Big[(1-C)\,u_f\Big]=0,\qquad(12)$$

$$C\,(\rho_p/\rho_f)\ \mathrm{Re}\,\delta\left\{\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial z}+v_p\frac{\partial u_p}{\partial r}\right\}=-C\,\frac{\partial p}{\partial z}+CS\,(u_f-u_p),\qquad(13)$$

$$C\,(\rho_p/\rho_f)\ \mathrm{Re}\,\delta\left\{\frac{\partial v_p}{\partial t}+u_p\frac{\partial v_p}{\partial z}+v_p\frac{\partial v_p}{\partial r}\right\}=-C\,\frac{\partial p}{\partial r}+CS\,\delta^2\,(v_f-v_p),\qquad(14)$$

$$\frac{1}{r}\ \frac{\partial}{\partial r}\,(r\,C\,v_p)+\frac{\partial}{\partial z}\,(C\,u_p)=0,\qquad(15)$$

where $\mathrm{Re}=\rho_f\,ca/\mu_o$ and $\delta=a/\lambda$ are Reynolds number and wave number, respectively.

Using the long wavelength approximation (i.e., $\delta\ll1$) of Shapiro et al. (1969), the equations describing the flow in the wave frame (moving with the speed, c) of reference, are obtained as

$$(1-C)\frac{dp}{dz}=(1-C)\frac{\mu}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})u_f+CS\,(u_p-u_f),\qquad(16)$$

$$C\,\frac{dp}{dz}=CS\,(u_f-u_p)\,,\qquad(17)$$

The non-dimensional boundary conditions are

$$u_f=-1\quad\text{at}\quad r=h=H/a=1+\phi\,\sin 2\pi\,z,$$

$$u_f=-1\quad\text{at}\quad r=a_1/a=\varepsilon.\qquad(18)$$

The expressions for the velocity profiles, u_f and u_p, obtained as the solution of equations (16) and (17), subject to the boundary conditions (18), are given as

$$u_f=-1-\frac{(dp/dz)}{4\mu(1-C)}\left\{h^2-r^2+(h^2-\varepsilon^2)\frac{\ln(r/h)}{\ln(h/\varepsilon)}\right\},\qquad(19)$$

$$u_p=-1-\frac{(dp/dz)}{4\mu(1-C)}\left\{h^2-r^2+(h^2-\varepsilon^2)\frac{\ln(r/h)}{\ln(h/\varepsilon)}+\frac{4\mu(1-C)}{S}\right\}.\qquad(20)$$

The instantaneous non-dimensional volumetric flow rate, q $(=q'/\pi a^2 c$; q' being the flow rate in wave frame of reference which is same as in laboratory frame of reference), is thus calculated as

$$q = 2\ (1-C)\ \int_{\varepsilon}^{h} r\,u_f\,dr + 2C \int_{\varepsilon}^{h} r\,u_p\,dr$$

$$= -\ (h^2 - \varepsilon^2) - \frac{h^2 - \varepsilon^2}{8\,\mu(1-C)} \frac{dp}{dz} \left\{ h^2 + \varepsilon^2 + \beta - \frac{h^2 - \varepsilon^2}{\ln(h/\varepsilon)} \right\}, \qquad (21)$$

or $\quad -\dfrac{dp}{dz} = \dfrac{8\,\mu(1-C)(q + h^2 - \varepsilon^2)}{(h^2 - \varepsilon^2)\,\chi(z)}$, $\qquad (22)$

with $\chi(z) = h^2 + \varepsilon^2 + \beta - (h^2 - \varepsilon^2)/\ln(h/\varepsilon)$, and $\beta = 8C(1-C)\mu/S$, a non-dimensional suspension parameter.

Following the report of Shapiro et al. (1969), one now determines the mean volumetric flow rate, Q over one period of the wave, as

$$Q = q + 1 + \frac{\phi^2}{2} - \varepsilon^2 . \qquad (23)$$

An application of the relation (23) into equation (22), yields the expression for the non-dimensional pressure drop, $\Delta p = p(0) - p(1)$, across one wavelength (which is same whether measured in wave or laboratory frame of reference), is thus calculated as

$$\Delta p = \int_0^1 (-\frac{dp}{dz})\,dz = 2\mu\,(1-C)\left\{ (Q - 1 - \varphi^2/2)\,I_1 + I_2 \right\}, \qquad (24)$$

with

$$I_1 = 4 \int_0^1 \frac{dz}{(h^2 - \varepsilon^2)\,\chi(z)} , \qquad I_2 = 4 \int_0^1 \frac{dz}{(1 - \varepsilon^2/h^2)\,\chi(z)} .$$

The friction force (at the wall) of the outer and inner tubes, $F_o (= F'_o / \pi\lambda c\mu_o)$ and $F_i (= F'_i / \pi\lambda c\mu_o)$ in their non-dimensional form are thus, obtained as

$$F_o = \int_0^1 h^2\,(-\frac{dp}{dz})\,dz$$

$$= 2\,\mu\,(1-C)\left\{ (Q - 1 + \varphi^2/2)\,I_2 + I_3 \right\} , \qquad (25)$$

$$F_i = \int_0^1 \varepsilon^2\,(-\frac{dp}{dz})\,dz$$

$$= 2\mu\,(1-C)\left\{ (Q - 1 + \varphi^2/2)I_4 + I_5 \right\} , \qquad (26)$$

where

$$I_3 = 4 \int_o^1 \frac{h^2 dz}{(1 - \varepsilon^2/h^2)\chi(z)} \quad , \quad I_4 = 4 \int_o^1 \frac{dz}{(h^2/\varepsilon^2 - 1)\chi(z)} \quad ,$$

$$I_5 = 4 \int_o^1 \frac{\varepsilon^2 dz}{(1 - \varepsilon^2/h^2)\chi(z)} \quad ,$$

The pressure-flow rate and friction force-flow rate relationships are thus obtained from equations (24)-(26) as

$$Q = 1 + \phi^2 / 2 - \frac{I_2}{I_1} + \frac{\Delta P}{2\mu(1-C)I_1} \quad , \tag{27}$$

$$F_o = 2\mu(1-C) \left\{ I_3 - \frac{I_2^2}{I_1} + \frac{I_2}{2\mu(1-C)I_1}\Delta P \right\} . \tag{28}$$

$$F_i = 2\mu(1-C) \left\{ I_5 - \frac{I_2 I_4}{I_1} + \frac{I_4}{2\mu(1-C)I_1}\Delta P \right\} . \tag{29}$$

The pressure rise ($-\Delta p$) for zero time-mean flow and the time mean flow for zero pressure rise which are of particular interest are given as

$$(-\Delta p)_{Q=0} = 2\mu (1-C) \left\{ (1 + \varphi^2/2)I_1 - I_2 \right\} , \tag{30}$$

$$(Q)_{\Delta p=0} = 1 + \varphi^2/2 - \frac{I_2}{I_1} . \tag{31}$$

In the limit, $\varepsilon \to 0$ (i.e., in the absence of the inner tube), one derives the expressions for the pressure drop, Δp and the friction force, F_o from equations (24) and (25) for the peristaltic induced flow of a particle-fluid mixture, as

$$\Delta p = 2\mu(1-C) \left\{ (Q - 1 - \varphi^2/2)L_1 + L_2 \right\} , \tag{32}$$

$$F_o = 2\mu(1-C) \left\{ (Q - 1 - \varphi^2/2)L_2 + L_3 \right\} , \tag{33}$$

With $\quad L_1 = 4 \int_o^1 \frac{dz}{h^4 + \beta h^2} \quad , \qquad L_2 = 4 \int_o^1 \frac{dz}{h^2 + \beta} \quad ,$

$$L_3 = 4 \int_o^1 \frac{dz}{1 + \beta/h^2} \quad .$$

Further, in the absence of the inner tube (i.e., under the limit, $\varepsilon \to 0$) and particle phase (i.e., C=0), the integrals involved in equations (24) and (25) become integrable in the closed form which yields the results of Shapiro et al (1969) as

$$\Delta p = \frac{8}{(1 - \phi^2)^{7/2}} \left\{ Q (1 + \frac{3}{2}\phi^2) + \frac{\phi^2}{4}(\phi^2 - 16) \right\} , \tag{34}$$

$$F_o = \frac{8}{(1 - \phi^2)^{3/2}} \left\{ Q - 1 - \varphi^2/2 + (1 - \varphi^2)^{3/2} \right\} . \tag{35}$$

NUMERICAL RESULTS AND DISCUSSION

To observe the effects of the various parameters involved, particularly, the particle concentration, C, the catheter size, ε and the amplitude ratio, ϕ on the results obtained above, computer codes are developed for the numerical evaluations of the analytical results derived in the study at a temperature of 25.5^{o}C. The parameter values are chosen as: a (tube radius) = 0.01 cm; C=0, 0.2, 0.4, and 0.6; ϕ = 0, 0.2, 0.4, 0.6, ε =0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6; a_o (radius of a particle) = 8μm and Q=0, 0.2, 0.4, 0.6, 0.8, and 1.0. The results of the present study under the limit, $\varepsilon\to0$ (in the absence of catheter), corresponds to the peristaltic flow of a particle-fluid mixture in circular cylindrical tube; C=0 (in the absence of particle phase), corresponds to the flow of a Newtonian viscous fluid through annular space of two circular tubes by means of peristaltic waves and ϕ=0 (no peristalsis), corresponds to the flow of a particulate suspension in the annulus of two concentric circular cylindrical tubes.
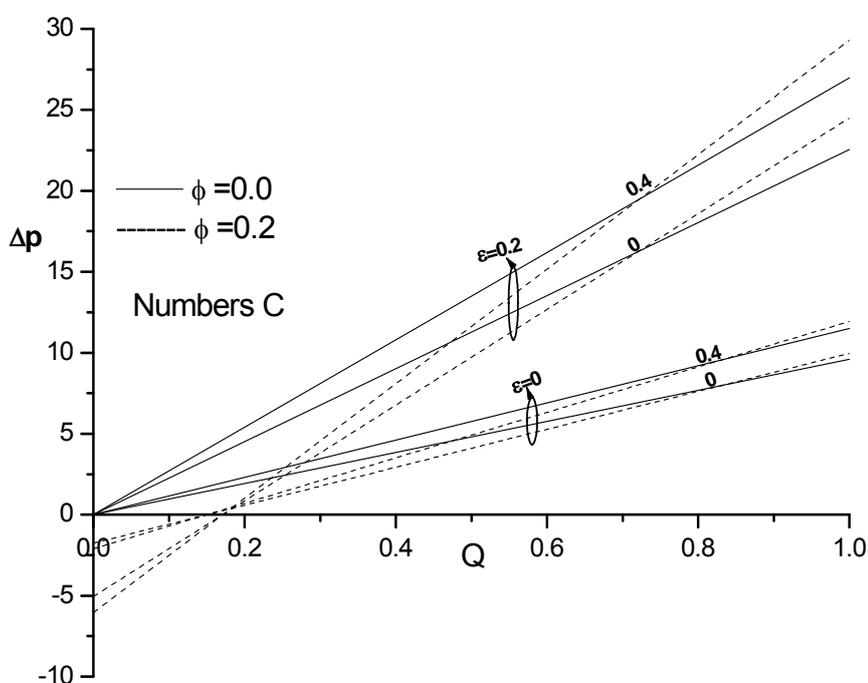

Fig. 2 Pressure-flow rate relationship for different C, ϕ and ε.

For any given set of parameters, a linear relationship between the flow and the pressure is observed (Fig.2). The pressure drop, Δp increases with flow rate, Q for other parameters given which in terns implies that an increase in the flow rate reduces the pressure rise (-Δp) and thus the maximum flow rate is achieved at zero pressure rise and the maximum pressure occurs at zero flow rate. Pressure drop, Δp increases with the catheter size, ε for any given flow rate, Q and the amplitude ratio, ϕ. The flow characteristics, Δp increases with particle concentration, C for any given flow rate, Q in both the catheterized and uncatheterized tubes. (Fig.2). One observes that the pressure drop, Δp decreases indefinitely with increasing amplitude ratio, ϕ for any given set of other parameters (Fig.3). Pressure drop, Δp assumes higher

magnitudes for larger flow rate, Q for small values of the amplitude ratio, ϕ but the property reverses for large values of ϕ (Fig. 3).For zero flow rate, the magnitude of the pressure drop, Δp increases with the particle concentration, C for any given value of the catheter size, ε and the amplitude ratio,ϕ , however, for zero flow rate, pressure drop, Δp decreases with increasing particle concentration, C in both the catheterized and uncatheterized tubes (Fig.4).
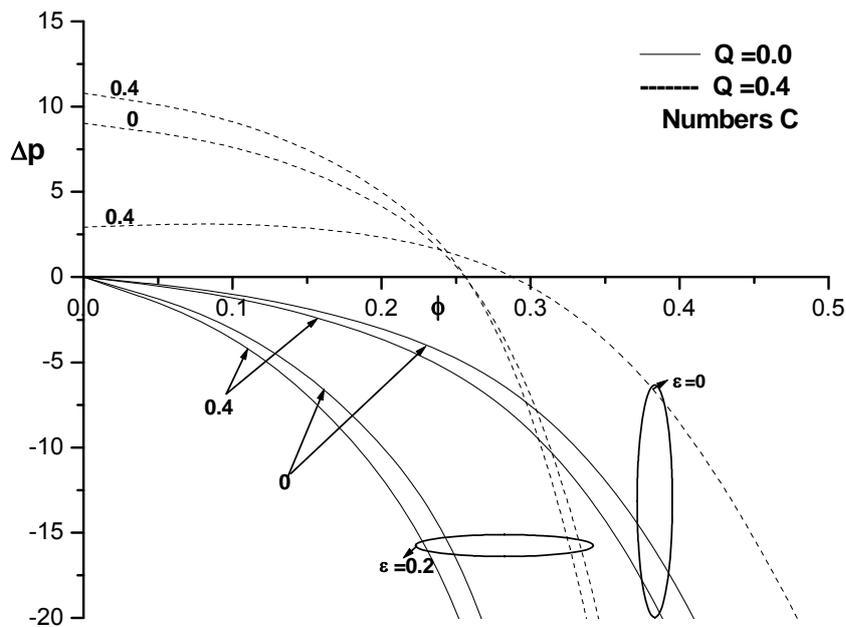
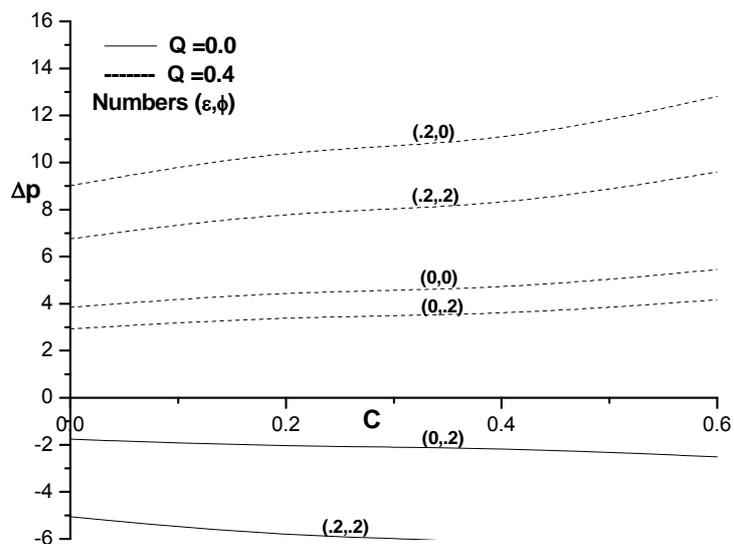Fig.3 Variation of Δp with ϕ for different Q, C and ε.

Fig.4 Variation of Δp with C for different Q, ϕ and ε.
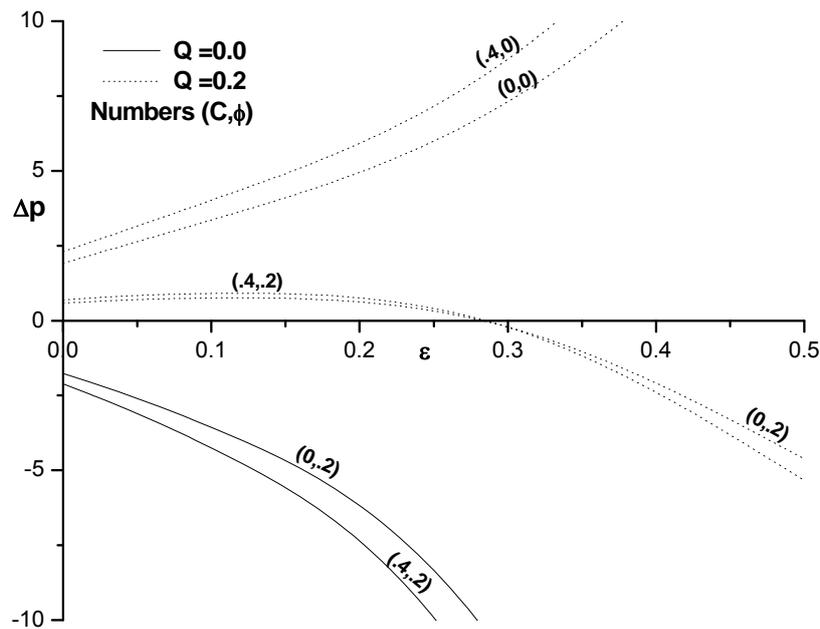
Fig.5 Variation of Δp with ε for different Q, ϕ and C.

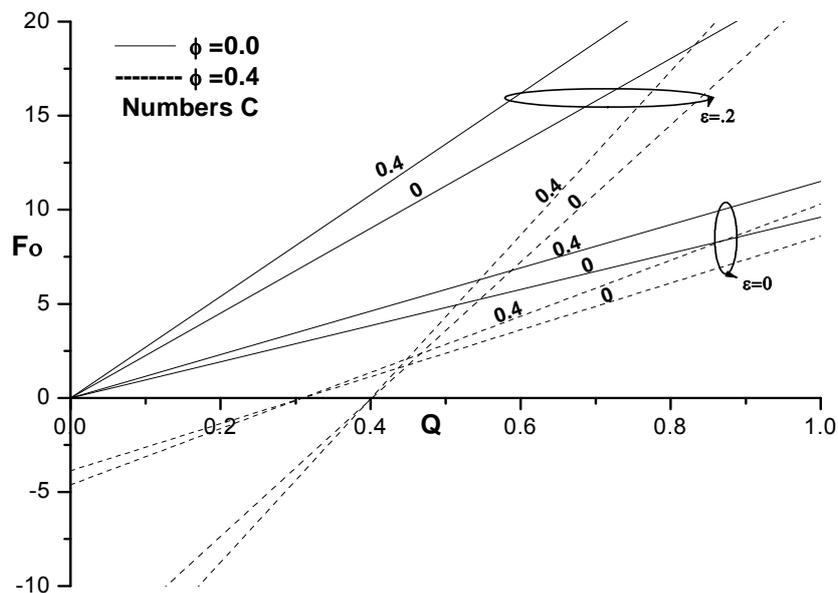

Fig.6 Variation of F_o with Q for different C, ϕ and ε.

In the absence of the peristaltic waves (i.e., ϕ =0), Δp increases with the catheter size, ε for any given particle concentration, C (Fig. 5). However, the flow characteristic, Δp decreases with increasing catheter size, ε for a given particle concentration C and non-zero values of the amplitude ratio, ϕ (Fig.5).
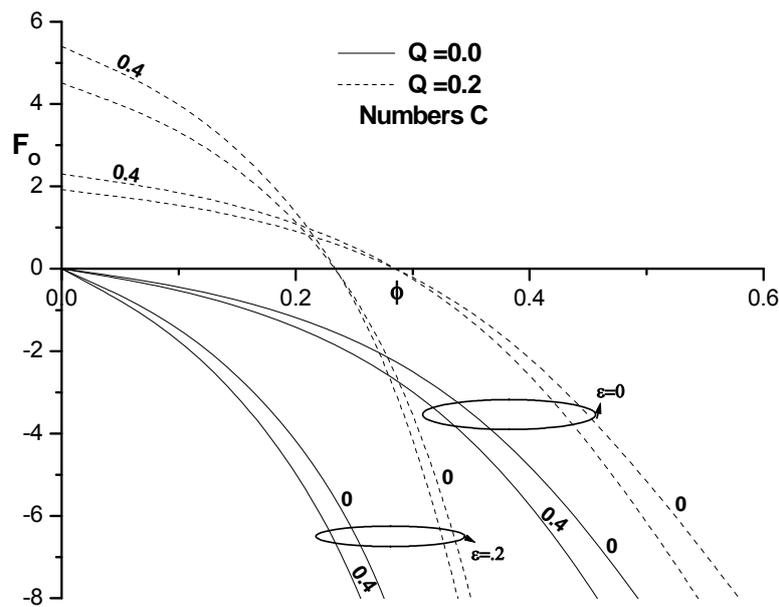
Fig.7 Variation of F_0 with ϕ for different Q, ϕ and ε.

The friction force at the tube wall, F_o increases with the flow rate, Q for any given values of C, ϕ and ε (Fig.6). For any given set of parameters, the flow characteristic, F_o decreases indefinitely with increasing amplitude ratio, ϕ (Fig.7). Friction force, F_o increases with particle concentration, C for any given values of Q, ϕ and ε (Fig.8). The flow characteristic, F_o decreases with increasing catheter size, ε for zero flow rate, Q (Fig.9), however, an inspection of Fig. 9 reveals that the variation in the magnitude of F_o possesses almost an opposite nature in the tube with peristaltic wave (ϕ =0) and without peristaltic wave ($\phi \neq 0$).

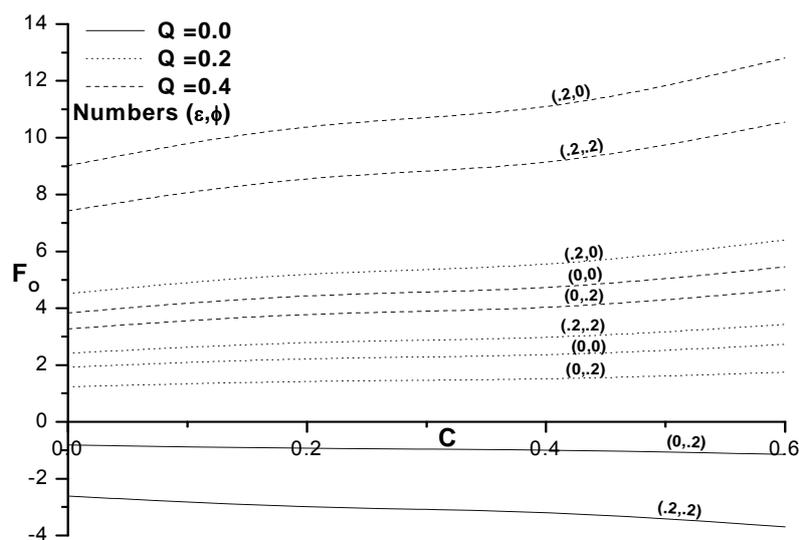Fig.8 Variation of F_o with C for different Q, ϕ and ε.

Fig.9 Variation of F_o with ε for different Q, ϕ and C.

Fig.10 Variation of F_o with Q for different C, ϕ and ε.

One observes that the friction force at the catheter wall, F_i too increases with the flow rate, Q (Fig.10). The flow characteristic, F_i decreases indefinitely with the increasing amplitude ratio, ϕ (Fig.11). For zero flow rate (i.e. Q=0), the friction force, F_i decreases with increasing particle concentration, C but increases with C for any non-zero flow rate, Q for any given values of catheter size, ε and amplitude ratio, ϕ (Fig.12). Friction force at the catheter wall, F_i decreases indefinitely with increasing catheter size, ε (Fig.13).

A comparison of the numerical results obtained for the pressure drop, Δp and the friction force at the tube wall, F_o reveals that the later possesses the characteristics

similar to the farmer with respect to any parameter (Figs. 2-9). One further notices that the magnitude of the friction force at the tube wall, F_o is much higher than the corresponding magnitude of the friction force at the catheter wall, F_i (Fig. 6-13).
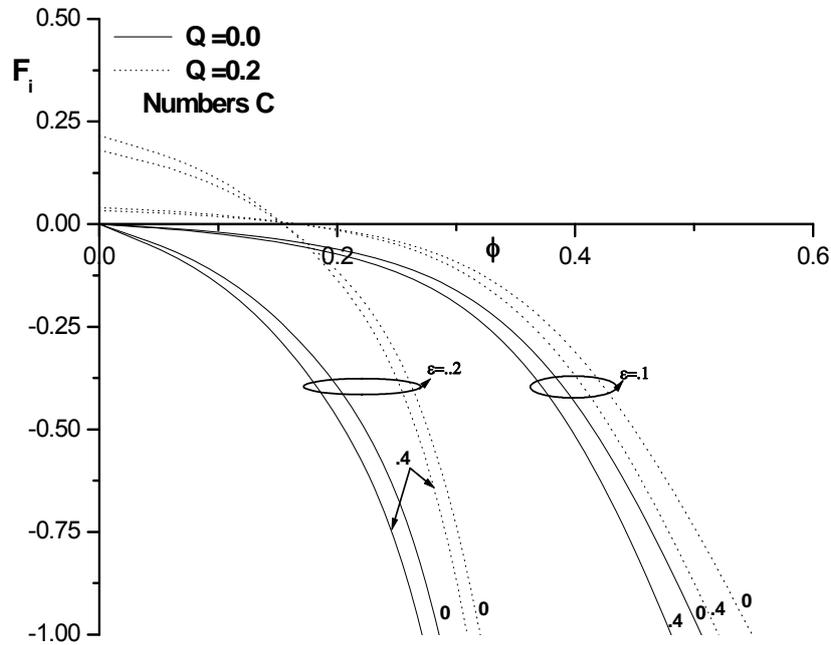
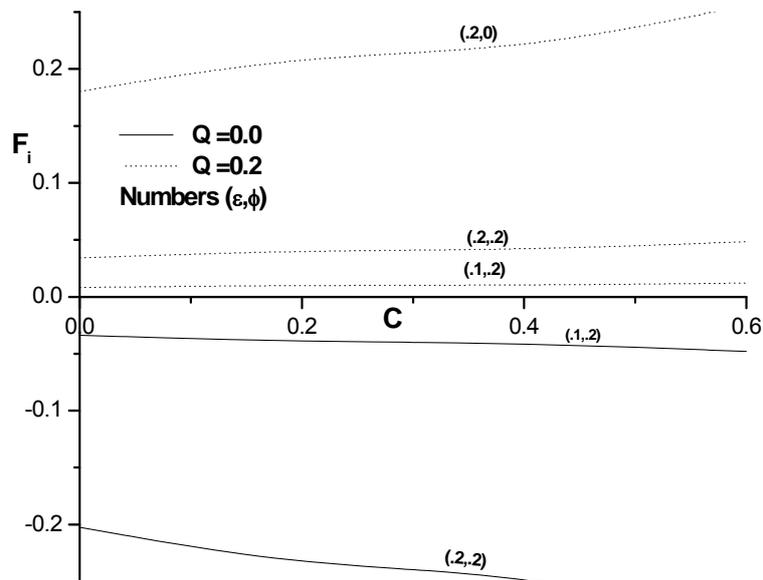Fig.11 Variation of F_i with ϕ for different Q, C and ε.

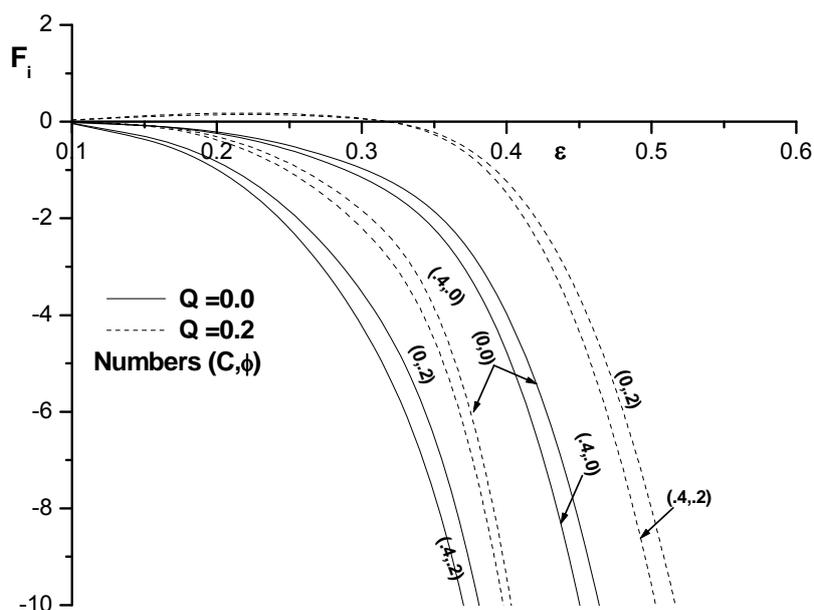Fig.12 Variation of F_i with C for different Q, ϕ and ε.

Fig.13 Variation of F_i with ε for different Q, ϕ and C.

CONCLUSIONS

To observe the effects of particle concentration and the amplitude ratio on flow behavior in a catheterized tube, the flow of a particle-fluid mixture in catheterized tube induced by peristaltic waves has been discussed. In view of the theoretical model used to conduct the study, it is obvious that the volume fraction density of the particle dominates the suspension property and therefore plays vital role in determination of the flow field, consequently a study based on particle concentration could be of practical use (Srivastava, 1996, 2007b). The informations that the pressure drop increases with the particle concentration for a given catheter size and amplitude ratio ϕ and also with the catheter size for any given particle concentration and the amplitude ratio seem to be of particular interest. Friction force at the tube wall assumes significantly higher magnitude than the corresponding friction force at the catheter wall which may be noted as another important result. The study enables one to observe the simultaneous effects of the particle concentration and the catheter size on peristaltic flow in a circular tube, seems to be the only one of its kind in the literature. In view of the theoretical model used to address the problem, it is believed that the present investigation may be applied to discuss the peristaltic pumping of blood through narrow catheterized arteries.

REFERENCES

1. Ali, N. and Hayat, T. Effects of an endoscope on peristaltic flow of a micropolar fluid. Mathematical and Computer Modeling 48, 721-733, 2008.

2. Back, L.H. Estimate mean flow resistance increase during coronary artery catheterization. J. Biomech. 27, 169-175, 1994.

3. Back, L.H., Kwack, E.Y. and Back, M.R. Flow rate pressure drop relation in coronary angioplasty: catheter obstruction effect. J. Biomech. Engng. 118, 83-89, 1996.

4. Batchelor, G.K. Diffusion of Particles with Homodynamic Interaction. J. Fluid Mech. 74, 1-29, 1976.

5. Batchelor, G.K. Transport Properties of Two-phase Materials with Random Structure. Ann. Rev. Fluid Mech.6, 225-227, 1974.

6. Brown, T.D. and Hung, T.K. Computational and experimental investigations of two-dimensional nonlinear peristaltic flows. J. Fluid Mech. 88, 249-272, 1977.

7. Charm, S.E. and Kurland, G.S. Blood Flow and Microcirculation. John Wiley, Newyork, 1974.

8. Drew, D.A. Stability of Stokes Layer of a Dusty Gas. Phys. Fluids 19, 2081-2084, 1979.

9. Hakeem, A.E., Naby, A.E. and Misery, A.M.E. Effects of an endoscope and generalized Newtonian fluid on peristaltic motion. Appl. Math. & Comput. 128, 19-35, 2002.

10. Hayat, T. and Ali, N. Peristaltic induced motion of a MHD third grade fluid in a deformable tube. Physica Acta 370, 225-239, 2006a.

11. Hayat, T. and Ali, N. On mechanism of peristaltic flows for power-law fluid. Physica Acta 371, 188-194, 2006b.

12. Hayat, T., Ali, N., Asghar, S. and Siddiqui, A.M. Exact peristaltic flow in tubes with an endoscope. Appl. Math. & Comput. 182, 359-368, 2006.

13. Hayat, T., Masood, S. and Asghar, S. Peristaltic transport of magneto hydrodynamic Johnson-Segalman fluid. Nonlinear Dynamics, 40, 375-385, 2005.

14. Hayat, T., Wang, Y., Hutter, K., Asghar, S. and Siddiqui, A.M. Peristaltic transport of an Oldroyd-B fluid in a planar channel. Math. Problems in Engng. 4, 347-376, 2004.

15. Hayat, T., Wang, Y., Siddiqui, A.M. and Hutter, K. Peristaltic motion of a Segalman fluid in a planar channel. Math. Problems in Engng. 1, 1-23, 2003.

16. Hayat, T., Wang, Y., Siddiqui, A.M., Hutter, K. and Asghar, S. Peristaltic transport of a third order fluid in a circular cylindrical tube. Math. Model and Methods in Appl. Sci. 12, 1671-1706, 2002.

17. Hayat, T., Ahmed, N. and Ali, N. Effects of an endoscope and magnetic field on peristalsis involving Jeffery fluid. Communication in Nonlinear Science and Numerical Simulation 13, 1581-1591, 2008a.

18. Hayat, T., and Ali, N. Slip effects on peristaltic transport of MHD fluid with variable viscosity. Physics Letters A 372, 1477-1489, 2008b.

19. Hung, T. K. and Brown, T. D. Solid particle motion in a two-dimensional peristaltic flows. J. Fluid Mech. 73, 77-96-1976.

20. Jaffrin, M.Y. and Shapiro, A.H. Peristaltic Pumping. Ann. Rev. Fluid Mech. 3, 13-36, 1971.

21. Latham, T.W. Fluid Motion in a Peristaltic Pump. M.S. Thesis, M.I.T. Mass, 1966.

22. MacDonald, D.A. Pulsatile flow in a catheterized artery. J. Biomech. 9, 239-249, 1986.

23. Medhavi, A. peristaltic pumping of a non-Newtonian fluid. Applic. and Appl. Math. 3, 137-148, 2008a.

24. Medhavi, A. and Singh, U. K. A two-layered suspension flow induced by peristaltic waves. Int. J. Fluid Mech. Res. 35, 258-272, 2008b.

25. Medhavi, A. and Singh, U. K. Peristaltic pumping of a two-layered particulate suspension in a circular cylindrical tube. Int. J. Theo. and Appl. Mech., In press, 2009.

26. Mekheimer, Kh. S. Nonlinear peristaltic transport through a porous medium in an inclined planner channel. J. Porous Media 6, 189-201, 2003.

27. Mekheimer, Kh., S. Peristaltic motion of a particle-fluid suspension in a planar channel. Int. J. Theo. Phys. 37, 2895-2920, 1998.

28. Misra, J.C. and Pandey, S.K. Peristaltic transport of blood in small vessels: study of a mathematical model. Comput. and Math. with Applic. 43, 1183-1193, 2002.

29. Misra, M. and Rao, A.R. Peristaltic transport of a Newtonian fluid in an asymmetric channel. ZAMP 54, 532-550, 2004.

30. Roos, R. and Lykoudis, P.S. The fluid mechanics of the ureter with an inserted catheter. J. Fluid Mech. 46, 615-625, 1970.

31. Sankar, D. S. and Hemlatha, K. Pulsatile flow of Herschel-Bulkley fluid through catheterized arteries: a mathematical model. Appl. Math. Modeling 31, 1497-1517, 2007.

32. Sarkar, A. and Jayaraman, G. Correction to flow rate- pressure drop relation in coronary angioplasty: steady streaming effect J. Biomech. 31, 781-791, 1998.

33. Shapiro, A.H., Jaffrin, M. Y. and Weinberg, S.L. Peristaltic pumping with long wavelength at low Reynolds number. J. Fluid Mech. 37, 799-825, 1969.

34. Siddiqui, A.M., Hayat, T. and Masood, K. Magnetic fluid influenced by peristaltic waves. J. Physical. Soc. Of Japan, 73, 2142-2147, 2004.

35. Srivastava, L.M. and Srivastava, V.P. Peristaltic Transport of a Particle-fluid suspension. Trans. ASME J. Biomech. Engng. 111, 157-165, 1989.

36. Srivastava, L.M. and Srivastava, V.P. Peristaltic transport of blood: Casson model II. J. Biomech. 17, 821-829, 1984.

37. Srivastava, V.P. Effects of an inserted endoscope on chyme movement in small intestine. Applc. and Appl. Math., 2, 79-91, 2007a.

38. Srivastava, V.P. and Saxena, M. A two-fluid model of non-Newtonian blood flow induced by peristaltic waves. Rheol. Acta 34, 406-414, 1995.

39. Srivastava, V.P. and Srivastava, L.M. Influence of wall elasticity and Poiseuille flow on peristaltic induced flow of a particle-fluid mixture. Int. J. Engng. Sci. 35, 1359-1361, 1997.

40. Srivastava, V.P. A theoretical model for blood flow in small vessels. Applc. and Appl. Math., 2, 51-65, 2007b.

41. Srivastava, V.P. Particle-fluid suspension flow induced by peristaltic waves in a circular cylindrical tube. Bull. Cal. Math. Soc. 94, 167-184, 2002.

42. Srivastava, V.P. Particle-fluid suspension model of blood flow through stenotic vessels with applications. Int. J. Biomed. Comput. 38, 141-154, 1995.

43. Takabatake, S. and Ayakawa, K. Numerical study of two-dimensional peristaltic flows. J. Fluid Mech. 122, 439-465, 1982.

44. Tam, C.K. W. The drag on a cloud of spherical particles in low Reynolds number flow. J. Fluid Mech. 38, 537-546, 1969.