Peristaltic Induced Flow of a Particulate Suspension in a Non-uniform Tube

Amit Medhavi and U. K. Singh

Department of Mechanical Engineering, Kamla Nehru Institute of Technology, Sultanpur-228118, India.

Ajai S. Yadav

Department of Mathematics, SMS Institute of Technology, Sultanpur Road, Lucknow, India.

Abstract-The flow of a particle-fluid suspension in a non-uniform tube induced by sinusoidal peristaltic wave motion of the wall has been investigated. The equations governing the flow for both the fluid and particle phases have been solved and the expressions for the flow rate, pressure rise and friction force has been derived. It is shown that pressure rise increases with the particle concentration but decreases with increasing flow rate. In addition, the pressure rise increases indefinitely with amplitude ratio for any given set of other parameters. Pressure rise assumes considerably smaller magnitude in a non-uniform tube as compared to its corresponding magnitude in uniform diameter tube. Friction force possesses the characteristics opposite to the pressure rise for any given set of parameters.

Key Words- Particle concentration, amplitude ratio, flow rate, pressure rise, friction force.

INTRODUCTION

In both the physiological and mechanical situations, fluid transport by means of a progressive wave of area contraction or expansion along the wall of a distensible duct containing liquid or mixture, has been the subject of scientific and engineering research for over four decades since the first investigation of Latham (1966). Physiologists term the phenomenon of such transport as peristalsis. Peristaltic pumping has been found to be involved in many biological organs including the vasomotion of small blood vessels such as arterioles, venules and capillaries (Srivastava and Srivastava, 1984), besides its practical applications involving biomechanical systems such as heart-lung machine, finger and roller pumps, etc. Jaffrin and Shapiro (1971) explained the basic principles and clearly brought out the significance of various parameters governing the flow. A summary of most of the experimental and theoretical investigations, reported up to the year 1983, arranged according to the fluid, the Reynolds number, the wave number, the amplitude ratio and the wave shape, was presented in an excellent article by Srivastava and Srivastava (1984). Srivastava and Saxena (1995) have well referenced the important contributions to the topic between the years 1984 and 1994. Some of the recent years studies include the investigations of Srivastava and Srivastava (1997), Mekheimer et al. (1998), Muthu et al. (2001), Srivastava (2002), Misra and Pandey (2002), Hayat et al. (2002,2003,2004), Mekheimer(2003), Misra and Rao (2004), Hayat et al. (2005), Hayat and Ali (2006 a, b), Srivastava (2007), Medhavi and coworkers (2008 a,b), Hayat and Coworkers (2008 a,b), Ali and Hayat (2008),Medhavi and Singh (2009a,b) and a few others.

The study of particulate suspension is very useful in understanding of a number of diverse physical problems concerned with powder technology, fluidization, and transport of solid particles by liquid and liquid slurries in chemical and nuclear processing, and metalized liquid fuel slurries for rocketry. The sedimentation of particles in a liquid is of interest in many chemical engineering processes. Recently, interest has developed in applying the theory of particle-fluid suspension to physiological flows as it provides improved understanding of the subjects such as diffusion of protein, the rheology of blood, the swimming of microorganism, the particle deposition on respiratory tract, etc. A large number of researches on the subject are referenced in Srivastava and Srivastava (1989).

Peristaltic pumping of a particle-fluid mixture has been investigated by Hung and Brown (1976), Srivastava and coworker (1989, 1997, 2002), Mekheimer et al. (1998), Medhavi and Singh (2008 b; 2009a,b) and several others. Barring the few (Gupta and Seshadri, 1976; Srivastava and coworkers, 1982, 1983, 1985, 1988; Mekheimer, 2002, etc.), most of the studies in the literature have been conducted in uniform geometry only whereas it is well known that in most of the practical applications, the flow geometry is found to be non-uniform. With increasing interest in particulate suspension flow due to its applications to diverse physical problems, the present study is therefore devoted to study the flow of a particulate suspension in a non-uniform tube induced by sinusoidal peristaltic waves. In view of the theoretical model for blood flow applied in Srivastava and Srivastava (2009) to discuss the effect of hematocrit on flow characteristics in a catheterized artery, it is strongly believed that the study conducted here may be applied to discuss the peristaltic induced flow of blood in small vessels of varying diameter.

FORMULATION OF THE PROBLEM

Consider the flow of a particulate suspension through a diverging tube with a sinusoidal wave travelling down its wall. The geometry of the wall surface is described (Fig.1) as

$$H(z, t) = a(z) + b \sin \frac{2\pi}{\lambda}(z - ct), \qquad (1)$$

$$\text{with} \qquad a(z) = a_o + kz, \qquad (2)$$

where a(z) is the tube radius at any axial distance z from the inlet, a_o is radius of the tube at the inlet (i.e., at z=0), k (<<1) is a constant whose magnitude depends on the length of the tube, exit and inlet dimensions, b is the amplitude of the wave, λ is the wavelength, c is the wave propagation speed and t is the time.

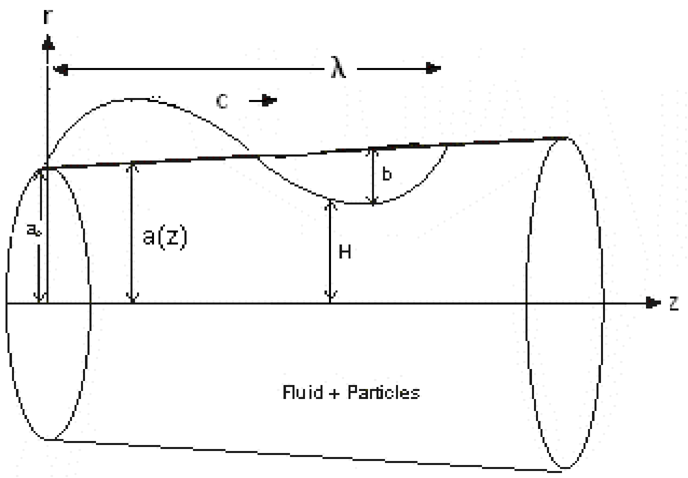

Fig. 1 Flow geometry of peristaltic waves in a non-uniform tube.

Using a continuum approach, the equations governing the linear momentum and the mass for both the fluid and particulate phases are expressed (Drew, 1979; Srivastava and coworkers, 1989, 2009) as

$$(1\text{-}C)\ \rho_f \left\{ \frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial z} + v_f \frac{\partial u_f}{\partial r} \right\}$$

$$= -(1\text{-}C)\frac{\partial p}{\partial z} + (1-C)\ \mu_s\ (C)\ \nabla^2 u_f \ + CS\,(u_p - u_f), \tag{3}$$

$$(1-C)\rho_f \left\{ \frac{\partial v_f}{\partial t} + u_f \frac{\partial v_f}{\partial z} + v_f \frac{\partial v_f}{\partial r} \right\} =$$
$$-(1-C)\frac{\partial p}{\partial r} + (1-C)\mu_s(C)(\nabla^2 - \frac{1}{r^2})v_f + CS(v_p - v_f), \tag{4}$$

$$\frac{1}{r}\frac{\partial}{\partial r}[r\,(1-C)\,v_f] + \frac{\partial}{\partial z}[(1-C)\,u_f] = 0. \tag{5}$$

$$C\ \rho_p \left\{ \frac{\partial u_p}{\partial t} + u_p \frac{\partial u_f}{\partial z} + v_p \frac{\partial u_p}{\partial r} \right\} = -\,C\frac{\partial p}{\partial z} + CS\,(u_f - u_p)\,, \tag{6}$$

$$C\ \rho_p \left\{ \frac{\partial v_p}{\partial t} + u_p \frac{\partial v_f}{\partial z} + v_p \frac{\partial v_p}{\partial r} \right\} = -\,C\frac{\partial p}{\partial r} + CS\,(v_f - v_p)\,, \tag{7}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\,C\,v_p\right]+\frac{\partial}{\partial z}\left[C\,u_p\right]=0, \tag{8}$$

where $\nabla^2 \equiv (1/r)\partial/\partial r\,(r\partial/\partial r)+\partial^2/\partial z^2$ is the two-dimensional Laplacian operator in cylindrical polar coordinate system; (r,z) are cylindrical polar coordinates with z measured along the tube axis and r perpendicular to the axis of the tube; (u_f, v_f) denotes the fluid phase and (u_p, v_p) particulate phase velocity components along (z, r) directions, respectively; ρ_f and ρ_p be the actual densities of the material constituting fluid and particulate phases, respectively ; C denotes the volume fraction of the particles, (1-C) ρ_f be the fluid phase and C ρ_p the particulate phase densities; p denotes the pressure; μ_s (C) $\cong$ μ_s be the suspension viscosity and S being the drag coefficient of interaction for the force exerted by one phase on the other. The concentration of the particles is assumed to be small enough so that the particle-particle impact due to the Brownian motion may be neglected. The volume fraction density of the particle, C is chosen to be a constant which is a good approximation for the low concentration of small particles (Batchelor; 1974, 1976).

The empirical relation for the suspension viscosity, μ_s and the expression of the drag coefficient of interaction, S for the present study are selected (Charm and Kurland, 1974; Tam, 1969; Srivastava and Srivastava, 2009) as

$$\mu_s = \mu_0/(1-mC),$$

$$m=0.070\exp\left[\,2.49C+(1107/T)\exp(-1.69C)\,\right], \tag{9}$$

$$S = 4.5\,\frac{\mu_o}{b_0^2}\,\eta(C),$$

$$\eta(C) = \frac{4+3(8C-3C^2)^{1/2}+3C}{(2-3C)^2}, \tag{10}$$

where μ_0 is the fluid (suspending medium) viscosity, b_o is the radius of a particle and T is measured in absolute scale of the temperature (K). The viscosity of the suspension expressed by the formula (9) is found to be accurate up to C=0.6 (Charm and Kurland, 1974; Medhavi and Singh, 2008b).

The introduction of the following dimensionless variables

$$z' = z/\lambda,\ r'= r/a_o,\ t'=ct/\lambda,\ (u'_f, u'_p) = (u_f, u_p)/c,$$

$$(v'_f,v'_p)= \lambda(v_f, v_p)/ca_o,\ p'=p\,a_0^2/\lambda c\mu_0,\ \mu = \mu_s/\mu_0,$$

$$S'=Sa_0^2/\mu_0,\ \delta = a_0/\lambda,\ R_e = c\rho_f\,a_0/\mu_0,$$

into equations (3)–(8), after dropping the primes, yields

$$(1\text{-C})\ \delta R_e \left\{ \frac{\partial u_f}{\partial t} + u_f \frac{\partial u_f}{\partial z} + v_f \frac{\partial u_f}{\partial r} \right\}$$

$$= -\,(1\text{-C})\ \frac{\partial p}{\partial z} + (1-C)\ \mu \left\{ \frac{\partial^2}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r} \frac{\delta^2\, \partial^2}{\partial z^2} \right\} u_f + CS\,(u_p - u_f), \qquad (11)$$

$$(1-C)\delta^3 R_e \left\{ \frac{\partial v_f}{\partial t} + u_f \frac{\partial v_f}{\partial z} + v_f \frac{\partial v_f}{\partial r} \right\} =$$

$$-(1-C)\frac{\partial p}{\partial r} + (1-C)\mu\,\delta^2 \left\{ \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \delta^2 \frac{\partial^2}{\partial z^2} - \frac{1}{r^2} \right\} v_f + CS\,\delta^2 (v_p - v_f) \qquad (12)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\,(1-C)\ v_f \right] + \frac{\partial}{\partial z}\left[(1-C)\,u_f \right] = 0\,, \qquad (13)$$

$$C\ (\rho_f\,/\,\rho_p)\,\delta R_e \left\{ \frac{\partial u_p}{\partial t} + u_p \frac{\partial u_p}{\partial z} + v_f \frac{\partial u_p}{\partial r} \right\} = -\,C\ \frac{\partial p}{\partial z} + \ CS\,(u_f - u_p), \qquad (14)$$

$$C\ (\rho_f\,/\,\rho_p)\,\delta^3 R_e \left\{ \frac{\partial v_p}{\partial t} + u_p \frac{\partial v_p}{\partial z} + v_f \frac{\partial v_p}{\partial r} \right\} = -\,C\ \frac{\partial p}{\partial r} + CS\ \delta^2\ (v_p - v_f), \quad (15)$$

$$\frac{1}{r}\ \frac{\partial}{\partial r}\left[r\ C\ v_p \right] + \frac{\partial}{\partial z}\left[C\,u_p \right] = 0\,, \qquad (16)$$

The Reynolds number, R_e is quite small when the wavelength is large (Jaffrin and Shapiro, 1971), and therefore, the inertial convective acceleration terms may be neglected in comparison to the viscous terms (Shapiro, et al. 1969). Thus, under the long wavelength approximation (i.e., $\delta \ll 1$), the equations (11)-(16) governing the flow in the laboratory frame of reference in its non-dimensional form reduce to

$$(1-C)\frac{dp}{dz} = (1-C)\frac{\mu}{r}\frac{\partial}{\partial r}(r\frac{\partial}{\partial r})u_f + CS(u_p - u_f)\,, \qquad (17)$$

$$C\frac{dp}{dz} = CS(u_f - u_p), \qquad (18)$$

$$\frac{dp}{dr} = 0, \qquad (19)$$

The non-dimensional boundary conditions are

$$u_f = 0 \text{, at} \quad r = h = H/a_o = 1 + \frac{\lambda K z}{a_0} + \phi \sin 2\pi (z-t), \tag{20}$$

$$\frac{\partial u_f}{\partial r} = \frac{\partial u_p}{\partial r} = 0 \quad \text{at} \quad r = 0. \tag{21}$$

ANALYSIS

The expressions for the velocity profiles, u_f and u_p obtained as the solution of equations (17) and (18), subject to the boundary conditions (20) and (21), are given as

$$u_f = -\frac{1}{4(1-C)\mu}\frac{dp}{dz}(h^2 - r^2), \tag{22}$$

$$u_p = -\frac{1}{4(1-C)\mu}\frac{dp}{dz}\left\{h^2 - r^2 + \frac{4(1-C)\mu}{S}\right\}, \tag{23}$$

The instantaneous volumetric flow rate, q (z, t) is thus calculated as

$$q(z,t) = 2\pi\int_o^h \left[(1-C)u_f + C\,u_p\right]dr$$

$$= -\frac{\pi}{8(1-C)\mu}\frac{dp}{dz}\left(h^4 + \beta h^2\right), \tag{24}$$

or

$$-\frac{dp}{dz} = \frac{8(1-C)\mu)\mu q(z,t)}{\pi h^2(h^2 + \beta)} \tag{25}$$

with $\beta = 8C(1-C)\mu/S$, a non-dimensional suspension parameter.

The pressure rise, $\Delta p_L(t)$ and the friction force at the wall, F_L (t) in a tube of length L in their non-dimensional form are given by

$$\Delta p_L(t) = \int_o^{L/\lambda}\left(\frac{dp}{dz}\right)dz, \tag{26}$$

$$F_L(t) = \int_o^{L/\lambda} h^2\left(-\frac{dp}{dz}\right)dz, \tag{27}$$

An application of equation (25) into equations (26) and (27), yields

$$\Delta p_L(t) = -8(1-C)\mu\int_0^{L/\lambda}\frac{q(z,t)}{\pi h^2(h^2+\beta)}dz \tag{28}$$

$$F_L(t) = 8(1-C)\mu \int_0^{L/\lambda} \frac{q(z,t)}{\pi(h^2+\beta)} dz \tag{29}$$

The expressions for the pressure rise and the friction force for a particle-fluid mixture in a uniform diameter tube may be obtained from relations (28) and (29) by setting K=0. In the absence of particle phase (i.e., C=0), the expressions given in equations (28) and (29) reduce to the results obtained in Gupta and Seshadri (1976) for a Newtonian fluid in non-uniform tube . Also, with K=0 and C=0 in equations (28) and (29), the results corresponding to Shapiro et al.(1969) in the laboratory frame of reference are derived from the present study.

NUMERICAL RESULTS AND DISCUSSION

In order to discuss the results of the study quantitatively, computer codes are now developed for the numerical evaluations of the analytical results for pressure rise, $\Delta p_L(t)$ and friction force $F_L(t)$, obtained in equations (28) and (29) respectively, for various parameter values at the temperature of 25.5^oC . The form of the instantaneous flow rate q (z, t) assumed to be periodic in (z-t) as (Gupta and Seshadri, 1976; Srivastava and Srivastava, 1988; Mekheimer, 2002)

$$\frac{q(z,t)}{\pi} = Q/\pi - \frac{\phi^2}{2} + \frac{2\lambda k z}{a_o} \phi \sin 2\pi(z-t) + 2\phi \sin 2\pi (z-t)$$

$$+ \phi^2 \operatorname{Sin}^2 2\pi (z-t), \tag{30}$$

where Q/π is the time average of the flow over one period of the wave. The above form of q (z, t) has been assumed in view of the fact that a constant value of q (z, t) gives $\Delta p_L(t)$ negative and consequently, there would be no pumping action in the tube wall. We now compute the dimensionless pressure rise, $\Delta p_L(t)$ and friction force, $F_L(t)$ over the tube length for various values of the dimensionless flow average, Q/π, amplitude ratio, ϕ and the particle concentration, C, using the form of q (z, t) given in equation (30). The average rise in pressure, Δp_L and friction force, F_L are then evaluated by averaging $\Delta p_L(t)$ and $F_L(t)$, respectively, over one period of the wave. Now using the parameter values (Srivastava and Srivastava, 1984, 1988; Mekheimer, 2002)

$a_o = 0.01$ cm, $L=\lambda= 10$ cm, $k = 0.5$ $a_o/L = 0.0005$,

the integrals involved in equations (28) and (29) are evaluated numerically and some of the critical results obtained are displayed graphically in Figs. 2-11.

The pressure rise, $\Delta p_L(t)$ decreases with increasing flow rate, Q for a given particle concentration, C but increases with the particle concentration, C for any given flow rate, Q (Fig. 2). The flow characteristic, $\Delta p_L(t)$ assumes much smaller magnitude in a non-uniform tube than its corresponding magnitude in a uniform tube (Figs. 2 and 3).

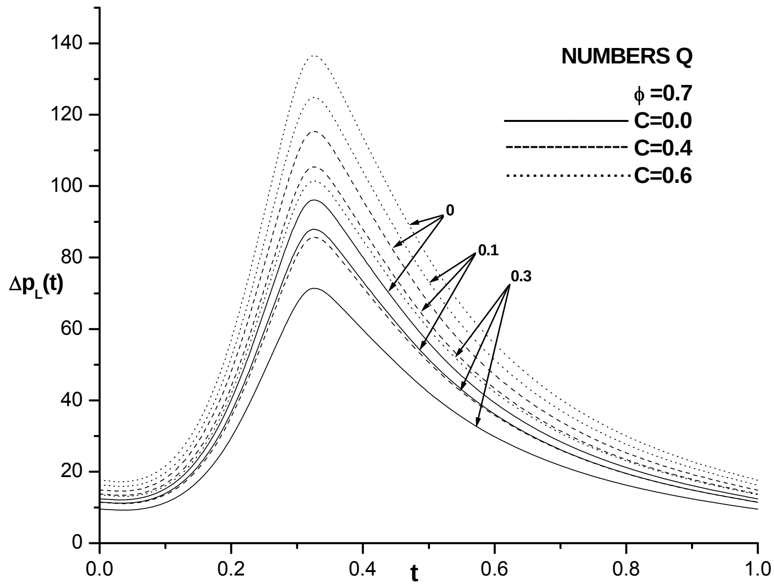

FIG.2 Variation of pressure rise,$\Delta p_L(t)$ with time, t for different Q and C in a non-uniform tube.

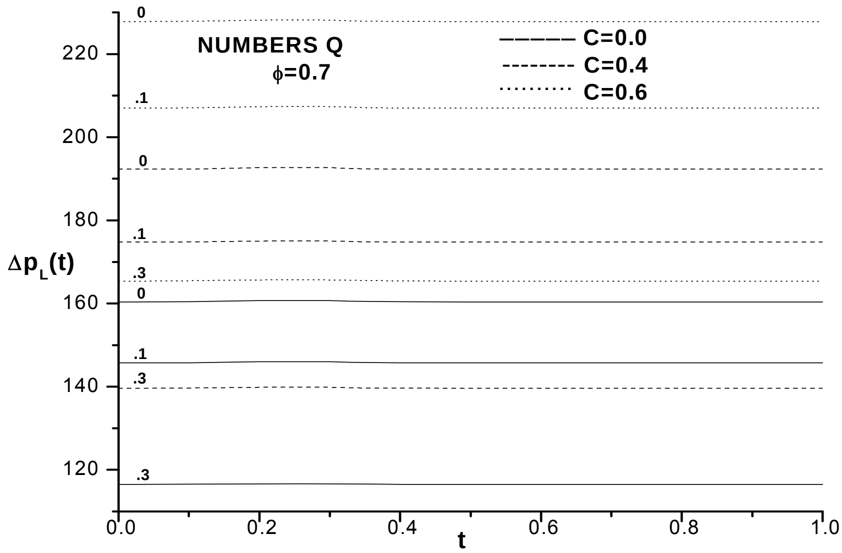

FIG.3 Variation of pressure rise, $\Delta p_L(t)$ with time, t for different Q and C in a uniform tube.

average pressure rise, $\Delta p_L(t)$ versus time averaged flow rate, Q has been shown in Fig.4 in a non- The The uniform tube which indicates a linear relationship between $\Delta p_L(t)$ and Q and thus the maximum flow rate is achieved at zero pressure rise and maximum pressure occurs at zero flow rate. The average pressure rise, Δp_L increases indefinitely with increasing

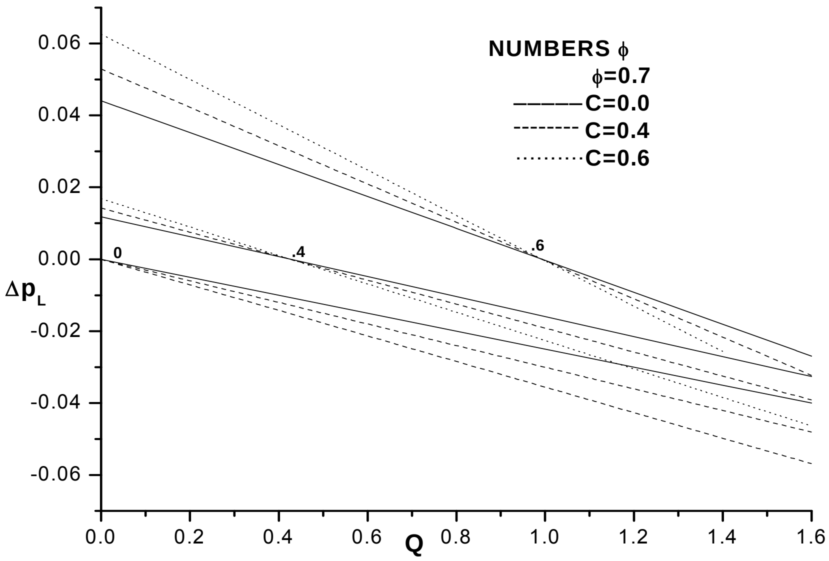

FIG.4 Pressure-flow rate relationship for different ϕ and C in a non-uniform tube.

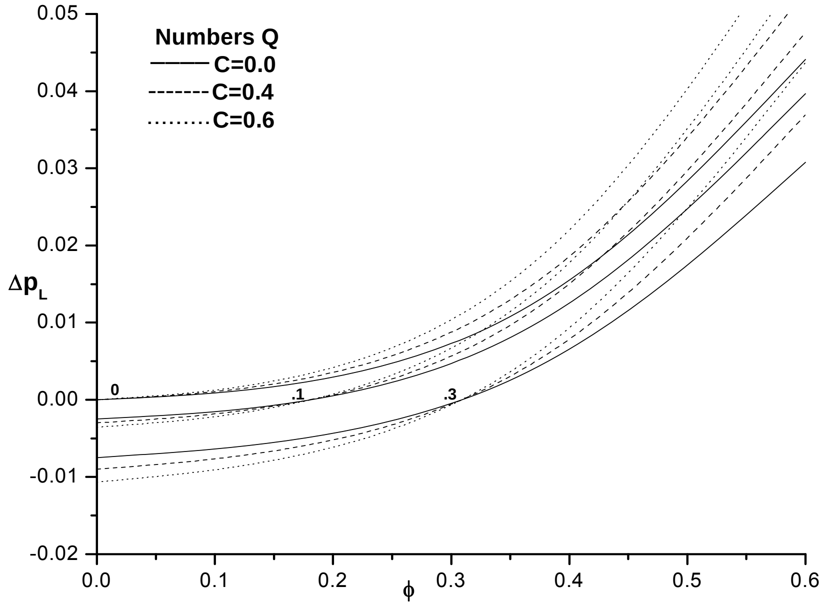

FIG.5 Variation of pressure rise, Δp_L with amplitude ratio, ϕ for different Q and C in a non-uniform tube.

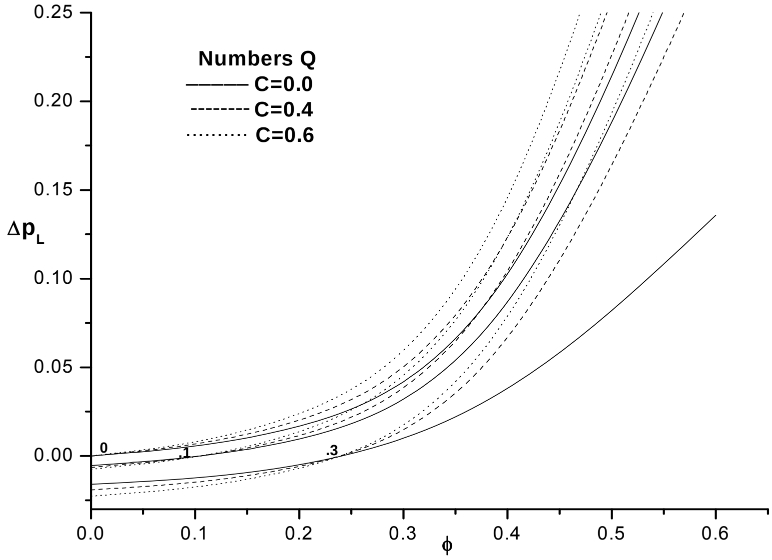

FIG.6 Variation of pressure rise, Δp_L for with amplitude ratio, ϕ for different Q and C in a uniform tube.

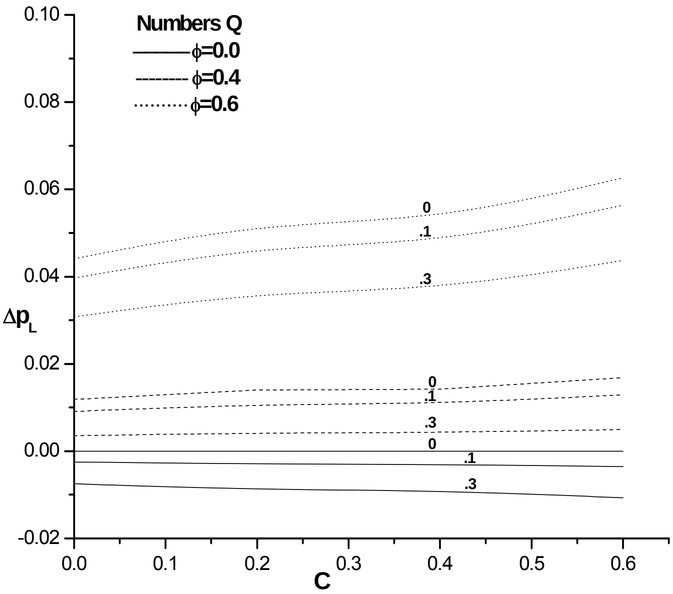

FIG.7 Variation of pressure rise, Δp_L with particle concentration, C for different Q and ϕ in a non-uniform tube.

amplitude ratio, ϕ for any given flow rate, Q and the particle concentration, C (Fig. 5). The flow characteristic, Δp_L assumes a very high asymptotic value at about $\phi = 0.6$ in both the uniform and non-uniform tube (Figs. 5 and 6).

However, Δp_L assumes considerable smaller magnitude in a non-uniform tube than its corresponding magnitude in a uniform diameter tube. The average pressure rise, Δp_L versus C have been plotted in Figs. 7 and 8 for non-uniform and uniform tubes, respectively. Δp_L is found to be increasing with the particle concentration, C at zero flow rate for a given amplitude ratio, ϕ. The nature in the variation of Δp_L with C is highly influenced with decreasing amplitude ratio, ϕ for any given non-zero value of the average flow rate, Q. One notices that Δp_L increases with amplitude ratio, ϕ for any given flow rate, Q (Figs.7 and 8).\

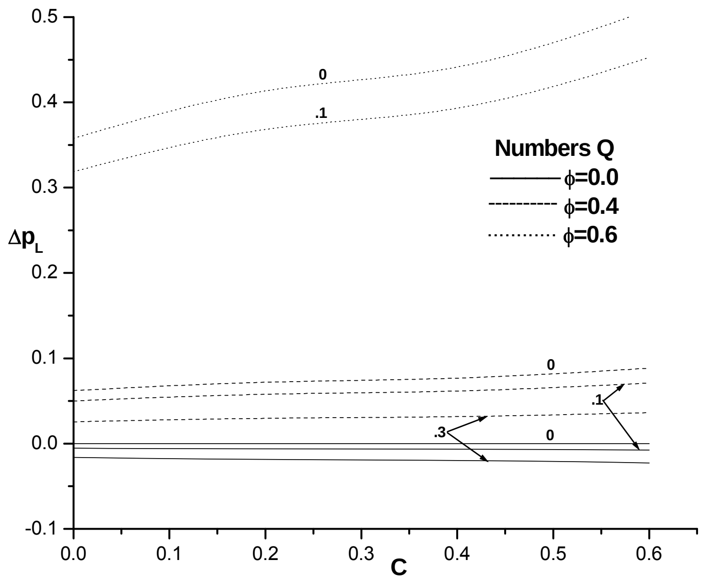

Fig. 8 Variation of pressure rise, Δp_L with particle concentration, C for different Q and ϕ in a uniform tube.

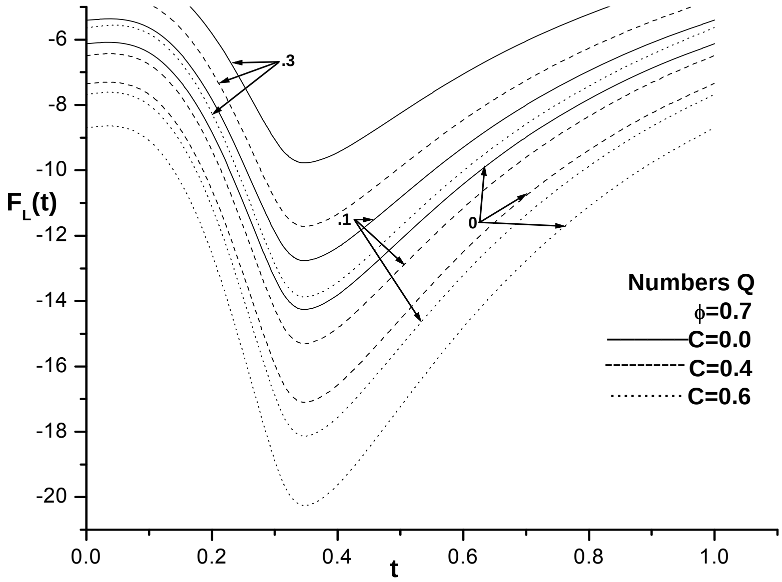

FIG.9 Variation of friction force, $F_L(t)$ with time, t for different Q and C in non-uniform tube.

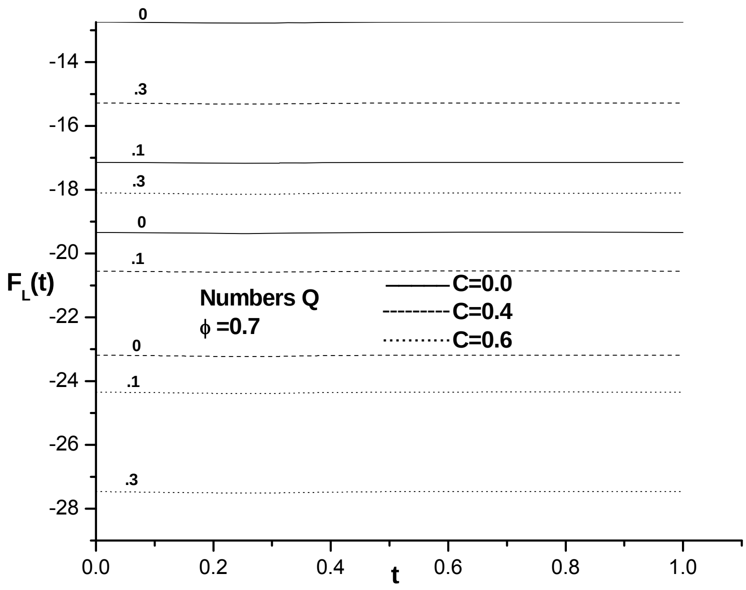

FIG.10 Variation of friction force, $F_L(t)$ with time, t for different Q and C in a uniform tube.

The friction force, $F_L(t)$ increases with the time averaged flow rate, Q for a given particle concentration, C, but decreases with increasing particle concentration, C for any given value of the flow rate, Q (Fig. 9). Friction force, $F_L(t)$ assumes higher magnitude in uniform tube than its

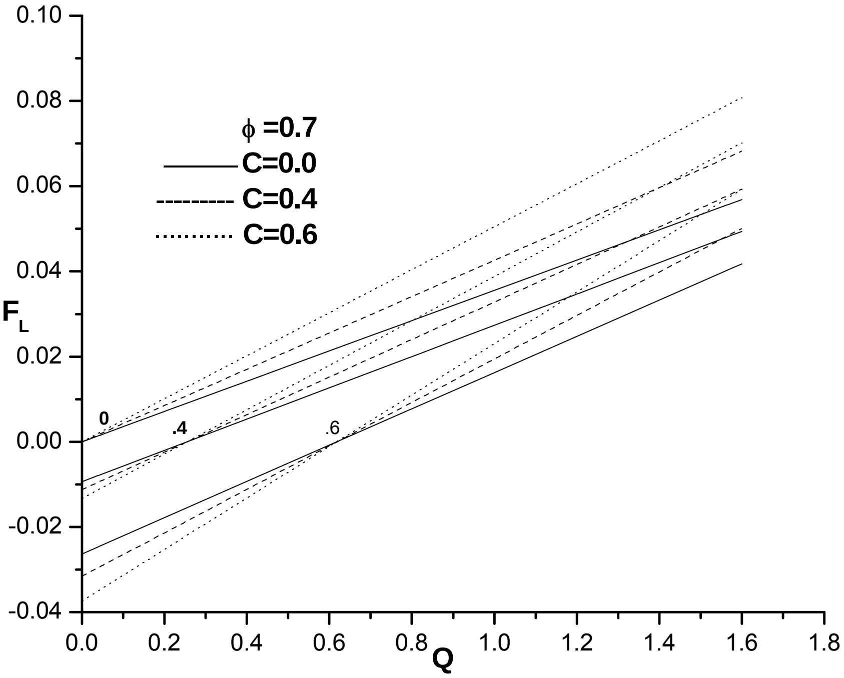

FIG.11 Friction force-flow rate relationship for different ϕ and C in a non uniform tube.

corresponding value in non-uniform tube (Fig. 9 and 10). It is noticed that the average friction force F_L increases with the flow rate, Q for any given particle concentration, C and the amplitude ratio, ϕ. A linear relationship between the average friction force, F_L and the flow rate, Q is noticed from Fig.11. An inspection of Figs. 2 and 9 reveals that the friction force, $F_L(t)$ possesses character opposite to that of the pressure rise, $\Delta p_L(t)$ for any given set of parameters. A similar conclusion is drawn in the case of their averaged values of pressure rise, Δp_L and friction force, F_L from Figs.4 and 11.

CONCLUSIONS

The flow of a particle-fluid mixture by means of peristaltic wave motion of the wall in a non-uniform tube has been discussed. The pressure rise decreases with the increasing flow rate for a given particle concentration but increases with the particle concentration for any given flow rate. A linear relationship exists between averaged pressure rise and the averaged flow rate. The friction force possesses the characteristics opposite to that of the pressure rise with respect to any parameter. The flow characteristics (pressure rise, averaged pressure rise, friction force, and averaged friction force) assume much smaller magnitude in a non-uniform tube than its corresponding value in uniform diameter tube. The flow equations considered to conduct the study corresponds to the macroscopic two-phase blood flow in narrow arteries. From the published literature, it is known that peristalsis contributes significantly towards the flow in small vessels. It is therefore strongly believed that the results of the present analysis may be applied to explain the peristaltic pumping of blood in small arteries.

REFERENCES

1. Ali, N. and Hayat, T. Effects of an endoscope on peristaltic flow of a micropolar fluid. Mathematical and Computer Modeling 48, 721-733, 2008.
2. Batchelor, G. K. Diffusion of particles with homodynamic interaction. J. Fluid Mech. 74, 1-29, 1976.
3. Batchelor, G. K. Transport properties of two-phase materials with random structure. Ann. Rev. Fluid Mech.6, 225-227, 1974.
4. Charm, S.E. and Kurland, G.S. Blood Flow and Micro Circulation. John Wiley, Newyork, 1974.
5. Drew, D. A. Stability of Stokes layer of a dusty gas. Phys. Fluids 19, 2081-2084, 1979.
6. Gupta, B. B. and Seshadri, V. Peristaltic pumping in non-uniform tubes. J. Biomech. 9, 105-109, 1976.
7. Hayat, T. and Ali, N. On mechanism of peristaltic flows for power-law fluid. Physica Acta 371, 188-194, 2006a.
8. Hayat, T. and Ali, N. Peristaltic induced motion of a MHD third grade fluid-in a deformable tube. Physica Acta 370, 225-239, 2006b.
9. Hayat, T., Ahmed, N. and Ali, N. Effects of an endoscope and magnetic field on peristalsis involving Jeffery fluid. Communication in Nonlinear Science and Numerical Simulation 13, 1581-1591, 2008a.
10. Hayat, T., and Ali, N. Slip effects on peristaltic transport of MHD fluid with variable viscosity. Physics Letters A 372, 1477-1489, 2008b.
11. Hayat, T., Masood, S and Asghar, S. Peristaltic transport of magnetohydro dynamic Johnson- Segalman fluid. Nonlinear Dynamics 40, 375-385, 2005.
12. Hayat, T., Wang, Y., Hutter, K., Asghar, S. and Siddiqui, A.M. Peristaltic transport of an Oldroyd-B fluid in a plannar channel. Math. Problems in Engng.4, 347-376, 2004.
13. Hayat, T., Wang, Y., Siddiqui, A.M. and Hutter, K. Peristaltic motion of a third order fluid in a circular cylindrical tube. Mathematical Models and Methods in Applied Sciences 12, 1671-1706, 2002.
14. Hayat, T., Wang, Y., Siddiqui, A.M. and Hutter, K. Peristaltic motion of a Segalman fluid in a plannar channel. Math. Problems in Engng.1, 1-23, 2003.
15. Hung, T.K. and Brown, T. D. Solid particle motion in a two-dimensional peristaltic flows. J. Fluid Mech. 73, 77-96, 1976.
16. Jaffrin, M. Y. and Shapiro, A. H. Peristaltic pumping. Ann. Rev. Fluid Mech. 3, 13-36, 1971.
17. Latham, T. W. Fluid Motion in Peristaltic Pump. M.S. Thesis, M.I.T. Cambridge, 1966.

18. Medhavi, A. peristaltic pumping of a non-Newtonian fluid. Applic. and Appl. Math. 3, 137-148, 2008a.

19. Medhavi, A. and Singh, U. K. A two-layered suspension flow induced by peristaltic waves. Int. J. Fluid Mech. Res. 35, 258-272, 2008b.

20. Medhavi, A and Singh, U. K. Peristaltic pumping of a two-layered particulate suspension in a circular cylindrical tube. Int. J. Theo. and Appl. Mech. 3, 265-280, 2009a.

21. Medhavi, A and Singh, U. K. Peristaltic pumping of a particulate suspension under long wavelength approximation. Int. J. of Dynamics of Fluids 5, 179-198, 2009b.

22. Mekheimer, Kh. S. Nonlinear peristaltic transport through a porous medium in an inclined planner channel. Journal of Porous Media 6, 189-201, 2003.

23. Mekheimer, Kh. S. Peristaltic transport of a couple stress fluid in a uniform and non-uniform channels. Biorheol. 39, 755-765, 2002.

24. MeKheimer, Kh. S., El-Shehawey, F.E. and Elaw, A.A. Peristaltic motion of a particle-fluid suspension in a planar channel. Int. J. Theo. Phys. 37, 2895-2920, 1998.

25. Misra, J. C. and Pandey, S. K. Peristaltic transport of blood in small vessels: study of a mathematical model. Computers and Mathematics with Applications 43, 1183-1193, 2002.

26. Misra, M. and Rao, A. R. Peristaltic transport of a Newtonian fluid in an asymmetric channel. ZAMP 54, 532-550, 2004.

27. Muthu, P., Kumar, B. V. R. and Chandra, P. Peristaltic motion in a circular cylindrical tubes: effects of wall properties. Ind. J. Pure and Applied Mathematics 32, 1317-1328, 2001.

28. Shapiro, A. H., Jaffrin, M. Y. and Weinberg, S.L. Peristaltic pumping with long wavelength at low Reynolds number. J. Fluid Mech. 37, 799-825, 1969.

29. Srivastava, V. P. and Srivastava, Rashmi. Particulate suspension blood flow through a narrow catheterized artery. Computers and Mathematics with Applications 58, 227-238, 2009.

30. Srivastava, L.M. and Srivastava, V.P. Peristaltic transport of a particle-fluid suspension. Trans. ASME J. Biomech. Engng. 111, 157-165, 1989.

31. Srivastava, L.M. and Srivastava, V.P. Peristaltic transport of a two-layered model of physiological fluid. J. Biomech. 15, 257-265, 1982.

32. Srivastava, L.M. and Srivastava, V.P. Peristaltic transport of a non-Newtonian fluid. Annals of Biomed. Engng. 13, 137-153, 1985.

33. Srivastava, L.M. and Srivastava, V.P. Peristaltic transport of a power-law fluid. Rheol. Acta 27, 428-433, 1988.

34. Srivastava, L.M. and Srivastava, V.P. Peristaltic transport of blood: Casson model II. J. Biomech. 17, 821-829, 1984.

35. Srivastava, L.M., Srivastava, V.P. and Sinha, S.N. Peristaltic transport of a physiological fluid. Part I, II & III. Biorheol. 20, 153-185, 1983.

36. Srivastava, V. P. Effects of an inserted endoscope on chyme movement in small intestine. Applications and Appl. Math., 2, 79-91, 2007.

37. Srivastava, V.P. and Saxena, M. A Two-fluid model of non-Newtonian blood flow induced by peristaltic waves. Rheol. Acta 34, 406-414, 1995

38. Srivastava, V.P. and Srivastava, L.M. Influence of wall elasticity and Poiseuille flow on peristaltic induced flow of a particle-fluid mixture. Int. J. Engng. Sci. 35, 1359-1361, 1997.

39. Srivastava, V.P. Particle-fluid suspension flow induced by peristaltic waves in a circular cylindrical tube. Bull. Cal. Math. Soc. 94, 167-184, 2002.

40. Tam, C.K. W. The drag on a cloud of spherical particles in low Reynolds number flow. J. Fluid Mech. 38, 537-546, 1969.

NOMENCLATURE

$H(z,t)$	shape of the peristaltic wave
a_0	radius of the tube at the inlet
$a(z)$	tube radius at axial distance z
λ	wavelength
t	time
c	wave velocity
C	volume fraction density of the particles
u_f, u_p	axial velocity component of fluid and particle phases, respectively
v_f, v_p	radial velocity component of fluid and particle phases, respectively
ρ_f, ρ_p	density of fluid and particles, respectively
(r,z)	two – dimensional cylindrical polar coordinates
μ_s	suspension viscosity
S	drag coefficient of interaction
μ_0	constant viscosity of suspending medium
b_0	radius of a particle
R_e	Reynolds number
δ	wave number
p	pressure
$\Delta p_L(t), \Delta p_L$	pressure rise, averaged pressure rise, respectively
$F_L(t), F_L$	friction force averaged friction force, respectively
$\frac{q(z,t)}{\pi}, \frac{Q}{\pi}$	volumetric flow rate, averaged flow rate