# A Two-Layered Flow Induced by Peristaltic Waves in a Catheterized Tube

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**Abstract-**The flow of a two-layered Newtonian fluid induced by peristaltic waves in a catheterized tube has been investigated. The expressions for the flow characteristics- the flow rate, the pressure drop and the friction forces at the tube and catheter wall are derived. It is found that the pressure drop increases with the flow rate but decreases with the increasing peripheral layer thickness and a linear relationship between pressure and flow exists. The pressure drop increases with the catheter size (radius) and assumes a high asymptotic at the catheter size more that fifty percent of the tube size. The friction forces at the tube and catheter wall posses characteristics similar to that of the pressure drop with respect to any parameter. However, friction force at catheter wall assumes much smaller magnitude than the corresponding value at the tube wall.
**Keywords**: Peripheral layer, pressure drop, friction force, catheter size, amplitude ratio.

## INTRODUCTION

The flow induced by peristaltic waves in the wall of the flexible tubes has been the subject of engineering and scientific research since the first investigation of Latham (1966). Physiologists term the phenomenon of such transport as peristalsis. It is a form of fluid transport that occurs when a progressive wave of area contraction or expansion propagates along the length of a distensible duct containing liquid or mixture. Besides, its practical applications involving biomechanical systems such as heart-lung machine, finger and roller pumps, peristaltic pumping has been found to be involved in many biological organs including the vasomotion of small blood vessels (Srivastava and Srivastava, 1984). Shapiro et al. (1969) and Jaffrin and Shapiro (1971) explained the basic principles of peristaltic pumping and brought out clearly the significance of the various parameters governing the flow. A summary of most of the theoretical and experimental investigations reported up to the year 1983; arranged according to the geometry, the fluid, the Reynolds number, the amplitude ratio and the wave shape; has been presented in an excellent article by Srivastava and Srivastava (1984). The important contributions between the years 1984 and 1994 are cited in Srivastava and Saxena (1995). The literature beyond this and of recent years include the investigations of Srivastava and Srivastava (1997), Mekheimer et al. (1998), Srivastava (2002), Misra and Pandey (2002), Hayat et al. (2002,2003,2004,2005), Mekheimer(2003), Misra and Rao (2004), Hayat et al. (2005), Hayat and Ali (2006a,b), Srivastava (2007), Hayat and Coworkers (2008a,b), Ali and Hayat (2008), Medhavi and coworkers (2008a,b; 2009, 2010), and a few others. Except the few (Shukla et al., 1980; Srivastava and Srivastava, 1984; Srivastava and Saxena, 1995; Brasseur et al., 1987; Rao et al.1995; Mishra and Pandey, 2002; Medhavi and Singh, 2008b, etc.), most of the studies conducted in the literature deal with the peristaltic flow problem of single-layered of a Newtonian or non-Newtonian fluid.

The study of flow through a catheterized tubes is of immense practical applications in physiology and engineering. The use of catheters has become standard tool for diagnosis and treatment in modern medicine. An inserted catheter in an artery increases the impedance and modifies the pressure distribution and alters the flow field. A brief review on the subject has recently been presented by Srivastava and Srivastava (2009). The geometrically similar problem of peristaltic pumping to study the effects of inserted catheter on ureteral flow was analyzed by Roos and Lykoudis (1970). A number of authors including Hakeem et al. (2002), Hayat and coworkers (2006, 2008a, b), Srivastava (2007), etc. have explained the effects of an endoscope on the flow behavior of chyme in gastrointestinal tract. It is known from the published literature that studies conducted so far have considered the flow of a single-layered fluid in a catheterized tube. It is however, regretted that no efforts, at least to the authors knowledge, has been made to study the flow of a two-layered fluid through a catheterized tube. With the above discussion in mind, an attempt is therefore made here in the present paper to study the peristaltic induced flow of a two-layered Newtonian fluid in a catheterized tube. Mathematical model corresponds to the flow of a two-layered fluid through an annulus. The outer layer (peripheral) is a Newtonian fluid of constant viscosity and the inner layer (core region) is also a Newtonian fluid (the viscosity of which may vary depending on the flow conditions). The study is aimed at possible application of peristaltic induced flow of blood (Saran and Popel, 2001) in catheterized small vessels and chyme in small intestine with an inserted endoscope (Srivastava, 2007).

## FORMULATION OF THE PROBLEM

Consider the axisymmetric flow of a two-layered fluid in a catheterized tube of radius a, consisting of a central core region of radius $a_1$ filled with a Newtonian fluid of viscosity $\mu_c$, and a peripheral layer of thickness a- $a_1$ filled with a Newtonian fluid of constant viscosity $\mu_{p.}$ The tube wall is assumed to be flexible and the flow is induced by a sinusoidal wave traveling down its wall. The catheter is assumed to be a co-axial rigid circular cylinder of radius $a_c$. The geometry of the wall surface is described (Fig. 1) as

$$H(z,t) = a + b \sin\frac{2\pi}{\lambda}(z - ct), \qquad (1)$$

where b is the wave amplitude, λ is the wavelength, c is wave propagation speed, z is the axial coordinate and t is the time.

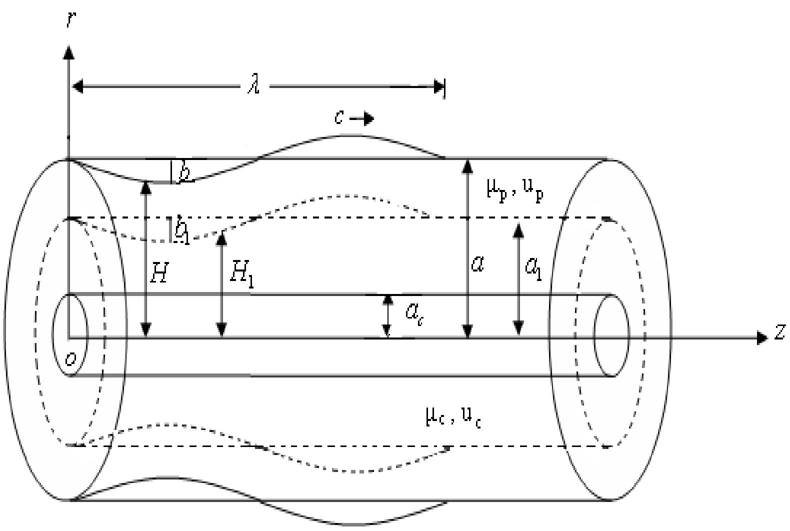



Fig.1 Flow geometry of a two-layered peristaltic pumping in a catheterized tube .

The equations governing the linear momentum and the conservation of mass for the fluid in the two regions (peripheral and core) using a continuum approach are expressed (Misra and Pandey, 2002; Sharan and Popel, 2001) as

$$\rho_p\left\{\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial z}+v_p\frac{\partial u_p}{\partial r}\right\}=-\frac{\partial p}{\partial z}+\mu_p\nabla^2 u_p,\quad H_1\le r\le H, \tag{2}$$

$$\rho_p\left\{\frac{\partial v_p}{\partial t}+u_p\frac{\partial v_p}{\partial z}+v_p\frac{\partial v_p}{\partial r}\right\}=-\frac{\partial p}{\partial r}+\mu_p\left(\nabla^2-\frac{1}{r^2}\right)u_p,\ H_1\le r\le H, \tag{3}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,v_p\right)+\frac{\partial}{\partial z}\left(r\,u_p\right)=0, \tag{4}$$

$$\rho_c\left\{\frac{\partial u_c}{\partial t}+u_c\frac{\partial u_c}{\partial z}+v_c\frac{\partial u_c}{\partial r}\right\}=-\frac{\partial p}{\partial z}+\mu_c\nabla^2 u_c,\quad a_c\le r\le H_1, \tag{5}$$

$$\rho_c\left\{\frac{\partial v_c}{\partial t}+u_c\frac{\partial v_c}{\partial z}+v_c\frac{\partial v_c}{\partial r}\right\}=-\frac{\partial p}{\partial r}+\mu_c\left(\nabla^2-\frac{1}{r^2}\right)u_c,\quad a_c\le r\le H_1, \tag{6}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,v_c\right)+\frac{\partial}{\partial z}\left(r\,u_c\right)=0, \tag{7}$$

where $\nabla^2=\partial/\partial r^2+(1/r)(\partial/\partial r)+\partial^2/\partial z^2$ is the two-dimensional Laplacian operator, $(u_p,v_p)$ and $(u_c,v_c)$ are (axial, radial) components of fluid velocity in peripheral and central regions, respectively, $(\rho_p,\rho_c)$ the fluid density in the (peripheral, central) regions, r is the radial coordinate and p is the pressure. In view of the argument stated in Misra and Pandey (2002) and Medhavi and Singh (2008b), one may now assume $H_1=a_1+b_1\sin 2\pi/\lambda\ (z-ct)$ in which $b_1$ is the amplitude of the interface wave.

Introducing the following dimensionless variables

$$r'=r/a,\ (u_p',u_c')=(u_p,u_c)/c,\ z'=z/\lambda,\ t'=ct/\lambda\ ,\ (v'_p,v'_c)=\lambda(v_p,v_c)/ac,$$

$$p'=a^2p/\lambda c\mu_c,\ \mu=\mu_p/\mu_c,\ (h,h_1)=(H,H_1)/a,=(1,\alpha)+(\phi,\phi_1)\sin 2\pi z,\ \varepsilon=a_c/a,$$
$$(\alpha,\ \phi,\ \phi_1)=(a_1,\ b,\ b_1)/a,$$

in to the equations (2) and (7), yields

$$\delta R_e\left\{\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial z}+v_p\frac{\partial u_p}{\partial r}\right\}=-\frac{\partial p}{\partial z}+\mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_p}{\partial r}\right)+\delta^2\frac{\partial^2 u_p}{\partial z^2}\right\},$$
$$h_1\le r\le h, \tag{8}$$

$$\delta R_e\left\{\frac{\partial v_p}{\partial t}+u_p\frac{\partial v_p}{\partial z}+v_p\frac{\partial v_p}{\partial r}\right\}=-\frac{\partial p}{\partial r}+\mu\delta^2\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_p}{\partial r}\right)-\frac{1}{r^2}+\delta^2\frac{\partial^2 v_p}{\partial z^2}\right\},$$
$$h_1\le r\le h, \tag{9}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,v_p\right)+\frac{\partial}{\partial z}\left(r\,u_p\right)=0, \tag{10}$$

$$\left(\rho_c/\rho_p\right)\delta R_e\left\{\frac{\partial u_c}{\partial t}+u_c\frac{\partial u_c}{\partial z}+v_c\frac{\partial u_c}{\partial r}\right\}=-\frac{\partial p}{\partial z}+\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_c}{\partial r}\right)+\delta^2\frac{\partial^2 u_c}{\partial z^2}\right\},\ \varepsilon\le r\le h_1, \tag{11}$$

$$\left(\rho_c/\rho_p\right)\delta R_e\left\{\frac{\partial v_c}{\partial t}+u_c\frac{\partial v_c}{\partial z}+v_c\frac{\partial v_c}{\partial r}\right\}=-\frac{\partial p}{\partial r}+\delta^2\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_c}{\partial r}\right)-\frac{1}{r^2}+\delta^2\frac{\partial^2 v_c}{\partial z^2}\right\},$$

$$\varepsilon\le r\le h_1, \tag{12}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\,v_c\right)+\frac{\partial}{\partial z}\left(r\,u_c\right)=0, \tag{13}$$

where $Re=\rho_p ca/\mu_c$ and $\delta=a/\lambda$ are Reynolds number and wave number, respectively.

The Reynolds number, *Re* is quite small when wavelength is large, and therefore, inertial convective acceleration terms may be neglected in comparison to viscous terms (Shapiro et. al, 1969; Jaffrin and Shapiro, 1971).Using thus the long wavelength approximation (i.e., $\delta<<1$) of Shapiro et al. (1969), and neglecting the inertial terms, the equations of motion in the moving frame of reference (moving with the speed of the wave) may be written as

$$\frac{dp}{dz}=\frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u_p,\ h_1\le r\le h, \tag{14}$$

$$\frac{dp}{dz}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u_c,\ \varepsilon\le r\le h_1. \tag{15}$$

The non-dimensional boundary conditions are

$$u_p=-1\ \text{at}\ r=h, \tag{16}$$

$$u_p=u_c\ \text{and}\ \tau_p=\tau_c\ ,\ \text{at}\ r=h_1=\alpha+\phi_1\sin 2\pi z, \tag{17}$$

$$u_p=-1\ \text{at}\ r=\varepsilon, \tag{18}$$

where $\tau_p=\mu_p\,\partial u_p/\partial r$ and $\tau_c=\mu_c\,\partial u_c/\partial r$ are shearing stresses of the peripheral and core the regions, respectively. The boundary conditions stated in eqns. (16)-(18) are the standard no slip condition on the tube wall and continuity of velocity and shear stress at the interface.

## ANALYSIS

The solution of the equations (14) and (15) subject to the boundary conditions (16)-(18), yields the expression for the velocity of peripheral and core fluids as

$$u_p=-1-\frac{1}{4\mu}\frac{dp}{dz}\left\{h^2-r^2+M\log(r/h)\right\}, \tag{8}$$

$$u_c=-1-\frac{1}{4}\frac{dp}{dz}\left\{\varepsilon^2-r^2+M\log(r/\varepsilon)\right\}, \tag{9}$$

with

$$M=\frac{-h^2+\mu\varepsilon^2+(1-\mu)h_1^2}{\log(h_1/h)-\mu\log(h_1/\varepsilon)}.$$

The non-dimensional instantaneous volume flow rate q (=q'/πa$^2$c, q' being the flux in the moving system which is same as in stationary system) is calculated as

$$q = \varepsilon^2 - h^2 - \frac{1}{8\mu}\frac{dp}{dz}\Big\{h^4 + (1-\mu)h_1^4 + 2(\mu\varepsilon^2 - h^2)h_1^2 - \mu\varepsilon^4 + M\left[\mu\varepsilon^2 - h^2 + (1-\mu)h_1^2 + 2\left(\mu\log\frac{h_1}{\varepsilon} - \log\frac{h_1}{h}\right)\right]\Big\}. \tag{10}$$

Using now the fact that the total flux is equal to the sum of the fluxes across the regions: $\varepsilon \le r \le h_1$ and $h_1 \le r \le h$ one derives the relations: $\phi_1 = \alpha\phi$ and $h_1 = \alpha h$ ( Shukla et al., 1980; Medhavi and Singh, 2008b). An application of these relations into eqn. (10), yields

$$-\frac{dp}{dz} = \frac{8\mu\left(q + h^2 - \varepsilon^2\right)}{\eta(\mu, \alpha, \varepsilon, h)}, \tag{11}$$

where

$$\eta \cong \eta(\mu, \alpha, \varepsilon, h) = \{1 + (1-\mu)\alpha^4 - 2\alpha^2\}h^4 + (2-\varepsilon^2)\mu\varepsilon^2 + N\left\{\mu\varepsilon^2 - \left[1 - (1-\mu)\alpha^2\right]h^2 + 2\left(\mu\log\frac{\alpha h}{\varepsilon} - \log\alpha\right)\alpha^2 h^2\right\},$$

$$N = \frac{\mu\varepsilon^2 - \{1 - (1-\mu)\alpha^2\}h^2}{\log\alpha - \mu\log(\alpha h/\varepsilon)}.$$

Following now Shapiro et al. (1969) and Medhavi (2010), the mean volume flow rate, Q over a period is determined as

$$Q = q + 1 + \phi^2/2 - \varepsilon^2. \tag{12}$$

The pressure drop, Δp = p(0) - p(1) across one wavelength is thus calculated as

$$\Delta p = \int_0^1 \left(-\frac{dp}{dz}\right) dz = 2\mu\left\{(Q - 1 - \phi^2/2)I_1 + I_2\right\}, \tag{13}$$

where $$I_1 = 4\int_0^1 \frac{Q - 1 - \phi^2/2}{\eta}\,dz,\quad I_2 = 4\int_0^1 \frac{h^2}{\eta}\,dz.$$

The friction force $F_a$ (= $F_a$'/ πλcμ$_p$; $F_a$' is the friction force at the tube wall in the stationary system which is same as in moving system) across one wave length is now obtained as

$$F_a = \int_0^1 h^2\left(-\frac{dp}{dz}\right) dz = 2\mu\left\{(Q - 1 - \phi^2/2)I_2 + I_3\right\}, \tag{14}$$

with $$I_3 = 4\int_0^1 \frac{h^4}{\eta}\,dz.$$

The friction force at the catheter wall, $F_c$ (= $F_c$' / πλcμ$_p$; $F_c$' being the friction force at the catheter wall in both the stationary and moving systems) across one wave length is now derived as

$$F_c = \int_0^1 \varepsilon^2 \left(-\frac{dp}{dz}\right) dz$$
$$= 2\mu\varepsilon^2 \left\{ \left(Q - 1 - \phi^2/2\right) I_1 + I_2 \right\}$$

(15)

From eqns. (13)-(15), one derives the following relations of particular interest as

$$Q = 1 + \phi^2/2 - \frac{I_2}{I_1} + \frac{\Delta p}{2\mu I_1}, \tag{16}$$

$$F_a = 2\mu\left[ I_3 - \frac{I_2^2}{I_1} + \frac{I_2}{2\mu I_1}\Delta p \right], \tag{17}$$

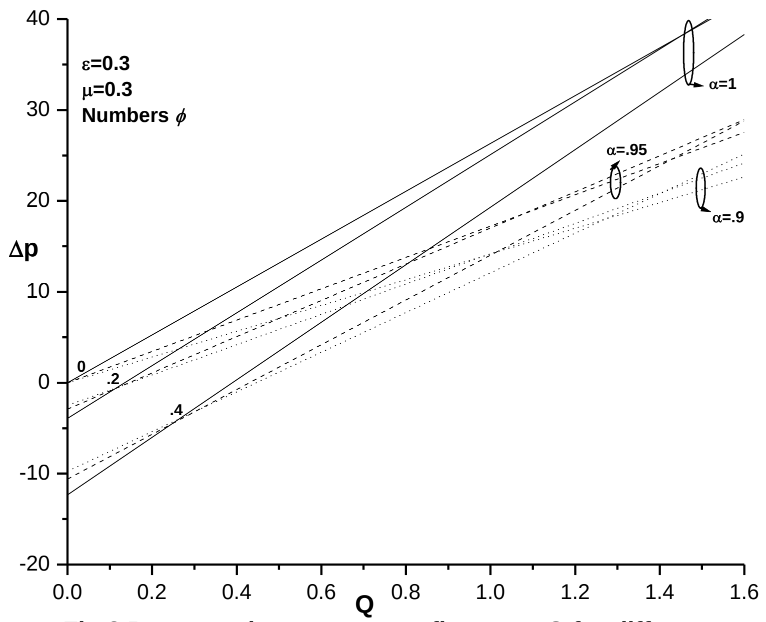
$$F_c = \varepsilon^2 \Delta p, \tag{18}$$

The pressure rise (-Δp) for zero time mean flow and the time mean flow for zero pressure rise which are of particular mechanical and physiological interest, are obtained from eqn. (16) as

$$\left(-\Delta p\right)_{Q=0} = 2\mu\left\{ \left(1 + \phi^2/2\right) I_1 - I_2 \right\}, \tag{19}$$

$$\left(Q\right)_{\Delta p=0} = 1 + \phi^2/2 - \frac{I_2}{I_1}. \tag{20}$$

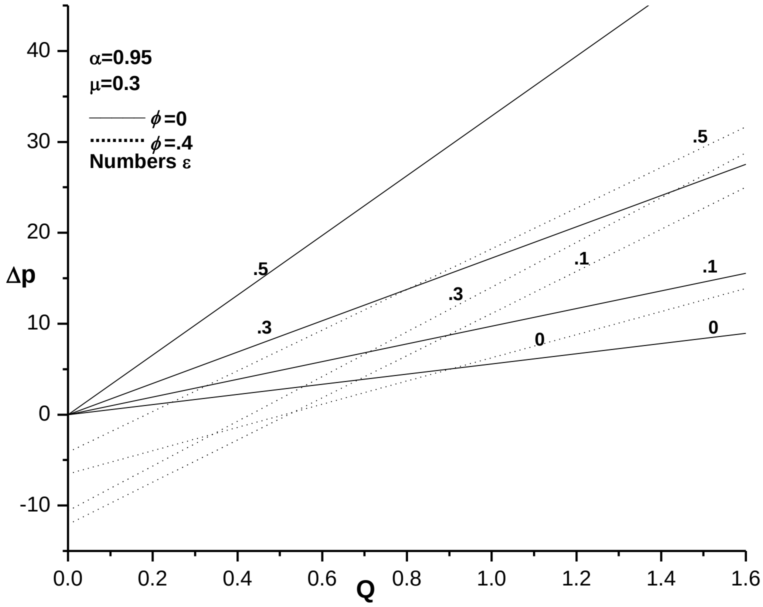
It is to note here that under the limit, $\varepsilon \to 0$ (no catheter); the results derived above yield the same results as obtained in Shukla et al. (1980). With $\alpha = 1, \mu = 1$, one derives the results obtained in Medhavi (2010) for a single-phase Newtonian viscous fluid. With $\alpha = 1, \mu = 1$ and $\varepsilon \to 0$, the results of Shapiro et al. (1980) are recovered from the present study.




**Fig,2 Pressure drop, Δp versus flow rate, Q for different α and φ.**

## NUMERICAL RESULTS AND DISCUSSIONS

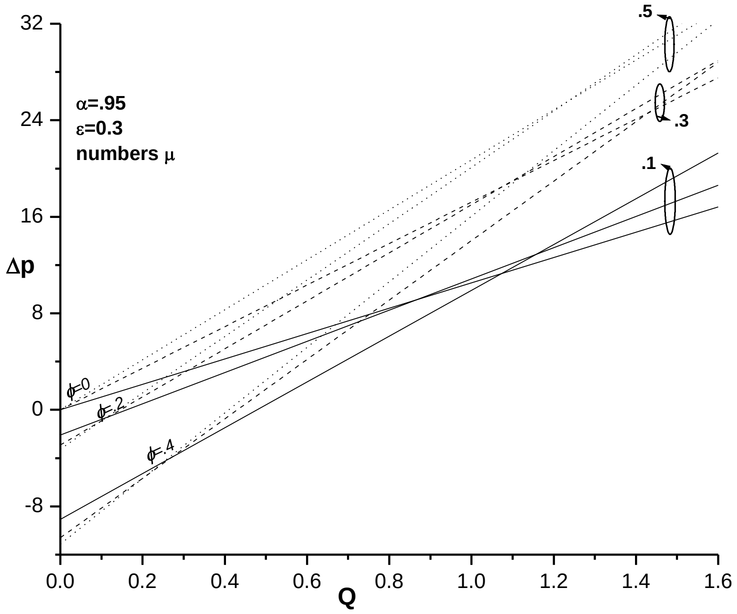
In order to discuss the results of the study quantitatively, computer codes are developed to evaluate analytical results obtained in eqns. (13) – (15) for various parameter values selected (Shukla et al., 1980; Mishra and Pandey, 2002; Medhavi, 2010) as: $\alpha$= 1, 0.95, 0.90; $\mu$= 0.1, 0.2, 0.3, 0.5, 1.0; $\varepsilon$ = 0, 0.1, 0.2, 0.3, 0.4, 0.5; $\phi$ = 0, 0.2, 0.4, 0.6; Q= 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6. Present study reduces to a two-layered flow in the absence of the catheter (Shukla et al., 1980), single-phase Newtonian viscous fluid in catheterized tube (Medhavi, 2010 in the absence




**Fig,3 Pressure drop, $\Delta$p versus flow rate, Q for different $\mu$ and $\phi$.**

of particle phase), single-phase Newtonian fluid in uncatheterized tube (Shapiro et al., 1969) for parameter values: $\varepsilon \rightarrow 0$, $\alpha$=1, $\mu$=1 in the absence of particle phase, respectively.

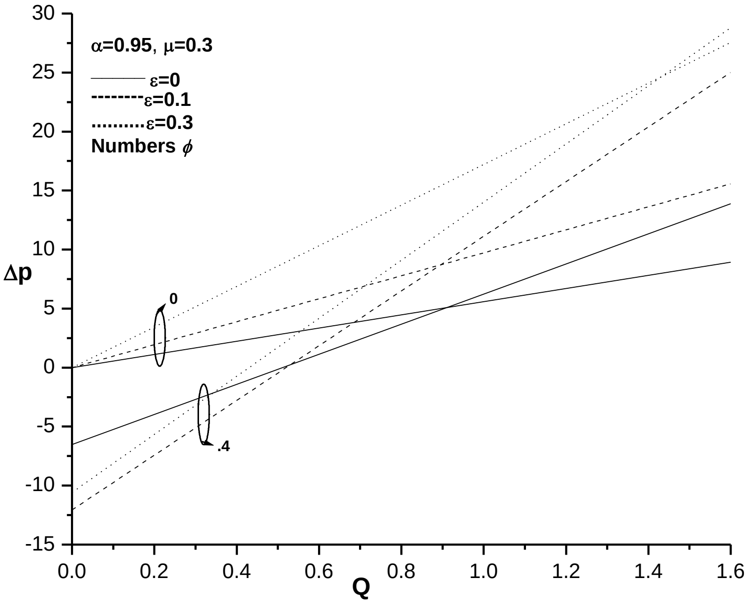
Pressure drop, $\Delta p$ increases with the flow rate, Q and a linear relationship between pressure and flow exhibits for any given set of parameters (Fig. 2). For any given flow rate, Q the flow characteristic, $\Delta p$ decreases with decreasing values of $\alpha$ (i.e., increasing peripheral layer thickness), however, depending on the magnitude of the non-zero amplitude ratio, $\phi$, this property reverses (Fig. 2). The pressure drop, $\Delta p$ increases with increasing value of the catheter size, $\varepsilon$ for any given flow rate, Q (Fig. 3).With other parameter fixed, $\Delta p$ increases




**Fig,4 Pressure drop, $\Delta p$ versus flow rate, Q for different $\mu$ and $\phi$.**

with the peripheral layer viscosity, $\mu$ for higher values of the flow rate, Q but the property reverses for small values of the flow rate, Q depending on the non-zero value of the amplitude ratio, $\phi$ (Fig. 4). The variation of the pressure drop, $\Delta p$ with respect to the peripheral layer viscosity, $\mu$ seems to have similar characteristics as with the peripheral layer thickness, $\alpha$ (Figs. 2 and 4).

The pressure drop, $\Delta p$ assumes higher magnitude for higher values of the catheter size, $\varepsilon$ for any given flow rate, Q in the absence of the peristaltic waves (i.e., $\phi = 0$). However, in the presence of peristaltic waves, $\Delta p$ assumes higher magnitude only for non-zero value of the catheter size, $\varepsilon$ (Fig. 5). The flow characteristic, $\Delta p$ decreases indefinitely with increasing amplitude ratio, $\phi$ for any given set of other parameters (Fig. 6). The pressure drop, $\Delta p$ decreases from its value in the absence of the catheter (i.e. $\varepsilon = 0$) and achieves a minimum value




**Fig,5 Pressure drop, Δp versus flow rate, Q for different ε and ϕ.**

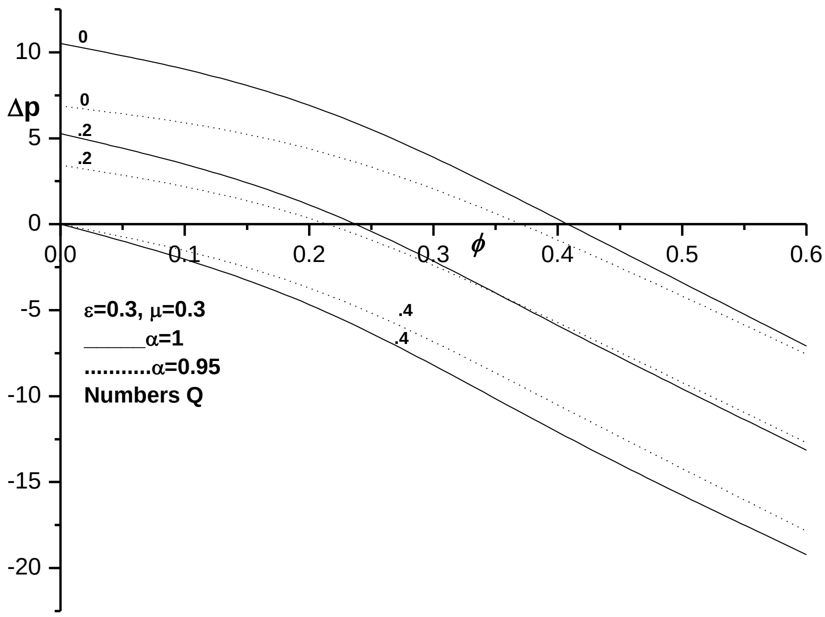



**Fig.6 Pressure drop, Δp versus ϕ for different α and Q.**

at about $\varepsilon = 0.2$, it then increases rapidly with increasing catheter size and assumes a high asymptotic magnitude when $\varepsilon > 0.5$ in the presence of the peristaltic waves (i.e., $\phi \neq 0$) but in the absence of the peristaltic waves (i.e., $\phi = 0$), $\Delta p$ increases with $\varepsilon$ (Fig. 7).

The friction force at the tube wall, $F_a$ increases with the flow rate, Q for any given set of parameters (Fig. 8). Also $F_a$ decreases with the amplitude ratio, $\phi$ (Fig.

9). One notices that the friction force at the catheter wall, $F_c$ possesses characteristics similar to that of $F_a$ with respect to any parameter but its magnitude is much lower than the corresponding magnitude of $F_a$ (Fig. 10). An inspection of Figs. 2, 8 and 10 reveals that $F_a$ and $F_c$ possess characteristics similar to that of the pressure drop, $\Delta p$ with respect to any parameter.

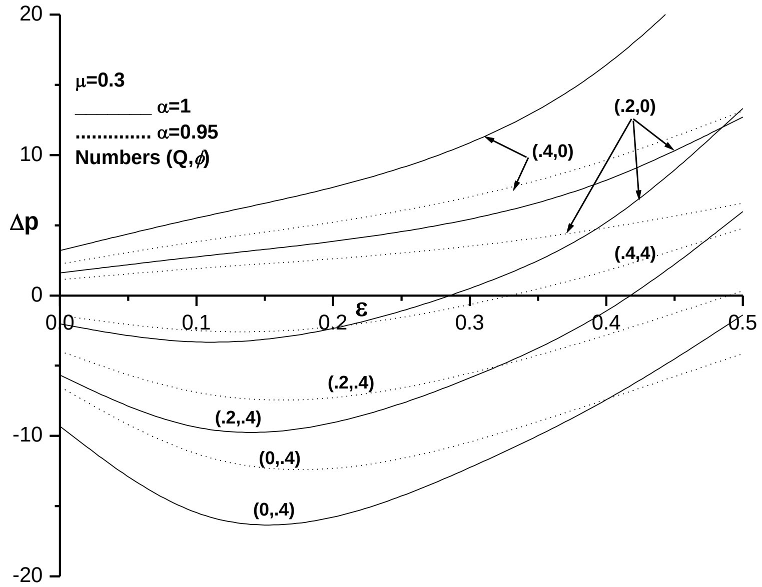
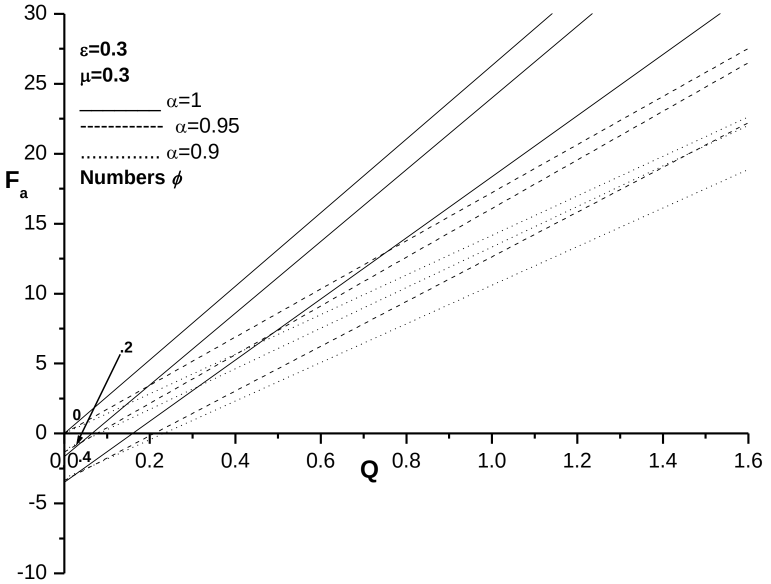
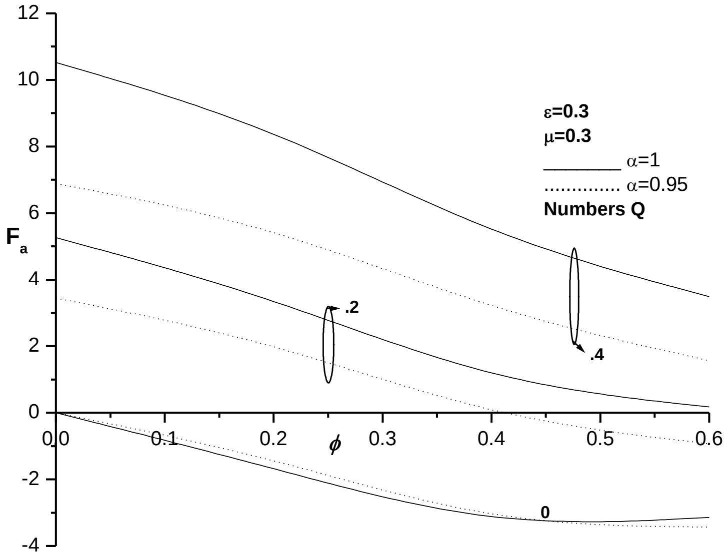



**Fig.7 Pressure drop, Δp versus ε for different α and ϕ.**



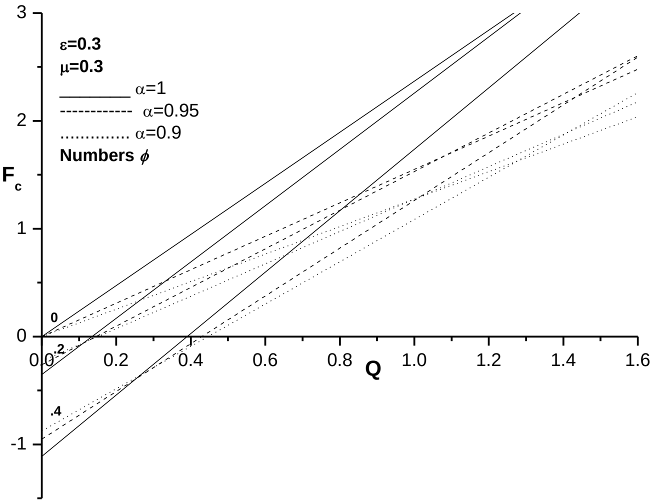

**Fig,8 Friction force at tube wall, $F_a$ versus flow rate, Q for different α and ϕ.**




**Fig,9 Friction force at tube wall, $F_a$ versus flow rate, $\phi$ for different α and Q.**

## CONCLUDING REMARKS

The flow induced by peristaltic waves of a two-layered Newtonian fluid in a catheterized tube has been addressed. The effects of the inserted catheter and the peripheral layer thickness has been observed simultaneously through out the analysis.The informations that the pressure drop increases with the catheter size and decreases with the parameter $\alpha$ (i.e., increasing peripheral layer thickness) may be noted as important observations. However, the study conducted above carries certain assumptions and approximations including the long wavelength




**Fig,10 Friction force at tube wall, $F_c$ versus flow rate, Q for different α and $\phi$.**

approximation and constant peripheral layer thickness. It is well known from the literature that the Reynolds number is quite small when the wavelength is long in most of the physical situations, particularly in biological organs (Misra and Pandey, 2002). This allows the inertia-free flow and fully developed flow equations (Shapiro et al., 1969). Further, some comments need to be made here regarding the shape of the interface. Misra and Pandey (2002) observed that the shape of the interface is not significantly affected when the viscosity of one of the layers is kept constant while the viscosity of the fluid in the other layer is varied. In view of the theoretical model used in the present work, the peripheral (outer) layer fluid viscosity, $\mu_p$ remains constant throughout and it is only the viscosity of the core fluid, $\mu_c$ may vary depending on the flow conditions. This justifies the use of the constant value of the parameter, $\alpha$ (i.e., constant peripheral layer thickness). In view of the theoretical model used (Sharan and Popel, 2001) to conduct the study, it is strongly believed that the findings of the work may be used to discuss the flow of blood through catheterized artery by means of the peristaltic waves.

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