BLOOD FLOW THROUGH A BELL-SHAPED STENOSIS IN CATHETERIZED ARTERIES

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Abstract

We consider the problem of blood flow through a catheterized artery in the presence of a bell shaped stenosis using Newtonian fluid model. The analytical expression for the blood flow characteristics, namely, the velocity profile, the flow rate, the impedance, the wall shear stress in the stenotic region and the shear stress at stenosis throat have been derived. The combined effect of stenosis and catheterization on flow characteristics is studied for different values of parameters.

Key words: Stenosis, Newtonian fluid, Impedance, Catheter, Shear Stress

INTRODUCTION

The frequently occurring cardiovascular disease, stenosis, is a medical term which means narrowing of anybody passage, tubes, or orifice (Young, 1979). It occurs due to the deposit of cholesterol, fatty substances, cellular waste products, calcium and fibrin in the inner lining of an artery. It is believed that stenosis caused by the impingement of extravascular masses or due to intravascular atherosclerotic plaque which develop at the wall of the artery and protrude into lumen. Regardless of the cause, it is well established that once an obstruction has developed, it results into significant changes in blood flow characteristics, pressure distribution, wall shear stress and the impedance (flow resistance). The flow through stenosed artery would provide the possibility of diagnosing the disease in early stages, by making treatment possible even before the stenosis becomes clinically significant. With the knowledge that the cardiovascular disease, stenosis is closely associated with the flow conditions and other hemodynamic factors, a large number of researchers including Young (1968), Young and Tsai (1973), Caro et al. (1978), Shukla et al. (1980), Ahmed and Giddens (1983), Sarkar and Jayaraman (1998), Pralhad and Schultz (2004), Jung et al. (2004), Liu et al. (2004), Srivastava and coworkers (2009, 2010, 2012), Mishra et al. (2006), Ponalagusamy (2007), Layek et al. (2009), Joshi et al. (2009), Mekheimer and El-Kot (2008), Tzirtzilakis (2008), Mandal and coworkers (2007), Misra and Verma (2007); Politis et al. (2008), Singh et al. (2010), Medhavi et al. (2012); Sankar, A. R. et al. (2013), Srivastav et al. (2013a, b, 2014a, b, c) and many others have addressed the stenotic development problems under various flow situations since the first investigation of Mann et al.(1938).

Being a suspension of corpuscles, at low shear rates blood in general behaves like a non-Newtonian fluid in small diameter tubes. The experimental observations of Cokelet (1972) and theoretical investigation of Haynes (1960) indicate that blood cannot be treated as a single-phase homogeneous viscous fluid while flowing through narrow arteries (of diameter $\leq$ 1000 μm). In large vessels, which include large cavities such as the ventricles and atria inside the myocardium as well as the large arteries and veins, the blood essentially behaves as a Newtonian fluid (Caro et al., 1978). No single model, Newtonian or non-Newtonian, can capture all the features of the blood complexities (Yilmaz et al., 2008) and hence different models are used to represent different characteristics of the blood rheology.

The use of catheter is of immense importance as a standard tool for diagnosis and treatment in a patient whose artery is affected adversely by the presence of stenosis. When a catheter is inserted into the stenosed artery the further increased impedance or frictional resistance to flow will alter velocity distribution. The mean flow resistance increase during catheterization in normal as well as in stenosed arteries has been studied by Back (1994) and Back et al. (1996). Sarkar and Jayaraman (1998) discussed the changed flow patterns of pulsatile blood flow in a catheterized stenosed artery. Sankar and Hemlatha (2007) studied the flow of Herschel-Bulkley fluid in a catheterized blood vessel. Srivastava et al. (2009, 2010) have discussed the macroscopic two-phase flow of blood in stenosed catheterized arteries.

Therefore, in this paper, the flow of the blood through the catheterized artery in the presence of a bell-shaped stenosis will be examined assuming that blood is represented by a Newtonian fluid. The wall in the vicinity of the stenosis is usually relatively solid when stenosis develops in the living vasculature. The artery length is considered large enough as compared to its radius so that the entrance, end and special wall effects can be neglected.

FORMULATION OF THE PROBLEM

Consider the axisymmetric flow of blood in a catheterized artery in the form of a circular cylindrical annular tube of radii, $R \approx R(z)$ and R_0 of the artery with and without stenosis respectively, R_c is the radius of the catheter, specified at the location as shown in Fig. 1. The geometry of the stenosis, assumed to be manifested in the arterial wall segment, is described (Srivastava et al., 2014c) as

$$\frac{R(z)}{R_0} = 1 - \frac{\delta}{R_0} \exp\left(\frac{-\omega^2 \varepsilon^2 z^2}{R_0^2}\right), \qquad |z| \leq L_0, \tag{1}$$

$$= 1; \qquad \text{otherwise} \tag{2}$$

where $2L_0$, 2L are the length of the stenosis and artery respectively, δ is the depth of stenosis at the throat and ω is a parametric constant, ε the relative length of the constriction, defined as the ratio of the radius to half length of the stenosis, i.e. $\varepsilon = R_0/L_0$. The artery length is assumed to be sufficiently large in comparison to its radius so that the end effects can be neglected.

Blood is assumed to be represented by a Newtonian fluid and following the report of Young (1968) and considering the axisymmetric, laminar, steady, one-dimensional flow of blood in an artery, the general constitutive equation in a mild stenosis case, under the conditions (Young, 1968; Srivastav et al. 2014c): $\delta / R_0 << 1$, $R_e(2\,\delta / L_0) << 1$ and $2\,R_0 / L_0 \sim O(1)$, may be stated as

$$\frac{dp}{dz} = \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)u, \quad (3)$$

where (r, z) are cylindrical polar coordinates system with z measured along the tube axis and r measured normal to the axis of the tube, R_e is the tube Reynolds number, p is the pressure and (u, μ) is the fluid (velocity, viscosity).

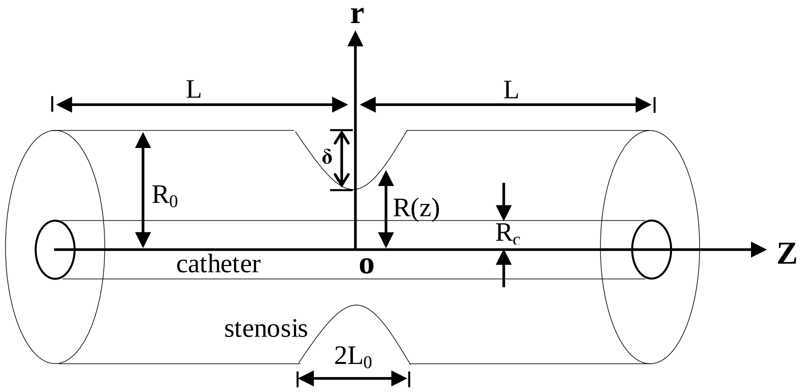

Figure 1. The geometry of a bell shaped stenosis in an artery

The boundary conditions are

$$u = 0 \ \text{ at } \ r = R, \quad (4)$$

$$u = 0 \ \text{ at } \ r = R_c, \quad (5)$$

Conditions (4) and (5) are the standard no slip conditions at the artery and catheter walls, respectively.

ANALYSIS

The expression for the velocity obtained as the solution of equation (3) subject to the boundary conditions (4) and (5), is given as

$$u = -\frac{R_0^2}{4\mu}\frac{dp}{dz}\left[\left(\frac{R}{R_0}\right)^2 - \left(\frac{r}{R_0}\right)^2 + \frac{\{(R/R_0)^2 - (R_c/R_0)^2\}}{\log(R/R_c)}\log\left(\frac{r}{R}\right)\right], \quad (6)$$

The flow flux, Q is thus calculated as

$$Q = 2\pi\int_{Rc}^{R} r\,u\,dr$$

$$= -\frac{\pi R_0^4\,\{(R/R_0)^2 - \varepsilon^2\}}{8\mu}\frac{dp}{dz}\left[\left(\frac{R}{R_0}\right)^2 + \varepsilon^2 - \frac{\{(R/R_0)^2 - \varepsilon^2\}}{\log(R/R_0\varepsilon)}\right], \quad (7)$$

where $\varepsilon = R_c/R_0$.

From equation (7), one now obtains

$$-\frac{dp}{dz}=\frac{8\mu Q}{\pi R_0^4}\varphi(z)\ , \tag{8}$$

with $\varphi(z)=1/F(z),\ F(z)=\{(R/R_0)^2-\varepsilon^2\}\left[\left(\frac{R}{R_0}\right)^2+\varepsilon^2-\frac{\{(R/R_0)^2-\varepsilon^2\}}{\log\{(R/R_0)/\varepsilon\}}\right].$

The pressure drop, $\Delta p\,(=p\text{ at }z=-L,-p\text{ at }z=L)$ across the stenosis in the tube of length, 2L is obtained as

$$\Delta p=\int_{-L}^{L}\left(-\frac{dp}{dz}\right)dz,$$

$$\Delta p=\frac{8\mu Q}{\pi R_0^4}\psi\ , \tag{9}$$

where $\psi=\int_{-L}^{-L_0}[\varphi(z)]_{R/R_0=1}dz+\int_{-L_0}^{L_0}[\varphi(z)]_{R/R_0\text{ from (1)}}dz+\int_{L_0}^{L}[\varphi(z)]_{R/R_0=1}dz,$

Using the definitions from the published literature, Srivastav R. K., (2014a), the expressions for the impedance (flow resistance), $\bar{\lambda}$ the wall shear stress in the stenotic region, $\bar{\tau}_w$ and the shear stress at stenosis throat, $\bar{\tau}_s$ are given as

$$\bar{\lambda}=\frac{\Delta p}{Q},\ \bar{\tau}_w=-\frac{R}{2}\left(\frac{dp}{dz}\right),\ \bar{\tau}_s=[\bar{\tau}_w]_{R/R_0=1-\delta/R_0},$$

Following now the reports of Young (1968) and Srivastav et al. (2013a); the expressions for impedance, λ, the wall shear stress, τ_w and shear stress at stenosis throat, τ_s, in their non-dimensional form are derived as

$$\lambda=\frac{(1-L_0/L)}{\eta}+\frac{1}{2L}\int_{-L_0}^{L_0}[\varphi(z)]_{R/R_0\text{ from (1)}}dz\ , \tag{10}$$

$$\tau_w=\frac{(R/R_0)}{\{(R/R_0)^2-\varepsilon^2\}[(R/R_0)^2+\varepsilon^2-\{(R/R_0)^2-\varepsilon^2\}/\log(R/R_0\varepsilon)]}, \tag{11}$$

$$\tau_s=[\tau_w]_{R/R_0=1-\delta/R_0}, \tag{12}$$

where $\eta=(1-\varepsilon^2)\{1+\varepsilon^2+(1-\varepsilon^2)/\log\varepsilon\}$, $\lambda=\bar{\lambda}/\lambda_0$, $(\tau_w,\tau_s)=(\bar{\tau}_w,\bar{\tau}_s)/\tau_0$, and λ_0 and τ_0 are the flow resistance and shear stress, respectively for a Newtonian fluid in a normal artery (no stenosis), and are given by

$$\lambda_0=\frac{16\mu L}{\pi R_0^4},\tau_0=\frac{4\mu Q}{\pi R_0^3}$$

NUMERICAL RESULTS AND DISCUSSION

To understand the physical characteristics of blood flow in a bell-shaped stenosed catheterized artery, the extensive quantitative analysis is performed through numerical computations and presented graphically. The various parameter values are selected from Srivastava and coworkers (2010) as: L_0 (cm) = 1; L (cm) = 1, 2, 5; δ/R_0 (non-dimensional stenosis height) = 0, 0.05, 0.10, 0.15, 0.20; ε (non-dimensional catheter radius) = 0, 0.1, 0.2, 0.3, 0.4, 0.5; for Eqs. (10)-(12). It is to note here that the present study corresponds to the flow in uncatheterized and normal (no stenosis) artery for parameter values ε =0 and $\delta/R_0 = 0$, respectively.

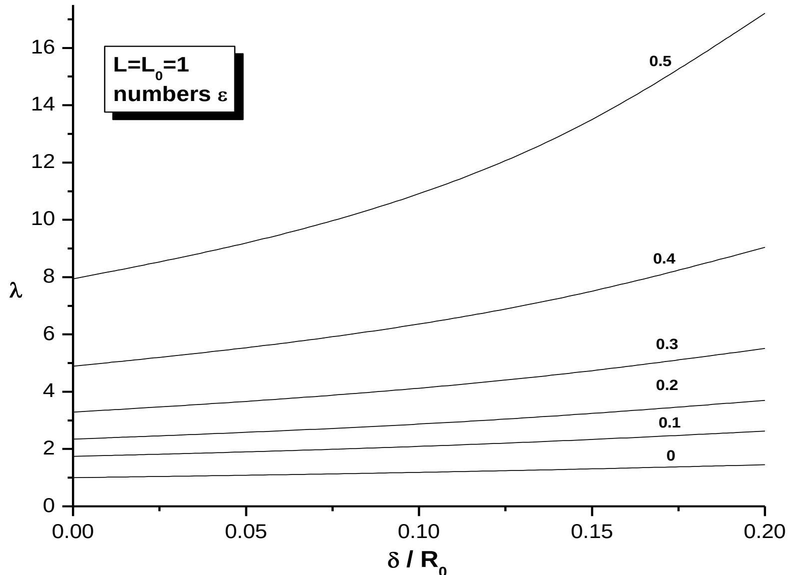

Fig.2 Impedance, λ versus stenosis height, δ/R_0 for different catheter size, ε.

Table 1. Variations of λ and τ_s with stenosis height, δ/R_0 and catheter size, ε .

$L_0/L = 1$						
δ/R_0	Impedance, λ and Shear stress, τ_s					
	$\varepsilon = 0$		$\varepsilon = 0.1$		$\varepsilon = 0.2$	
	λ	τ_s	λ	τ_s	λ	τ_s
0	1	1	1.74141	1.74141	2.34864	2.34864
0.05	1.08261	1.16635	1.89686	2.0626	2.57653	2.8318
0.10	1.1825	1.37174	2.0876	2.46762	2.86134	3.45749
0.15	1.30501	1.62833	2.32552	2.98569	3.22444	4.28262
0.20	1.45763	1.95313	2.62782	3.65913	3.69798	5.39393

Figure 2-4 depict the variations of resistance to flow, λ with stenosis height, δ/R_0, catheter size, ε and tube length, 2L. The impedance, λ increases with stenosis height, δ/R_0 for any given catheter size, ε and increases with the catheter size, ε for any given stenosis height, δ/R_0 (Fig.2). The flow resistance, λ steeply increases with the catheter size, ε (≤ 0.3) but increases rapidly with increasing the catheter size, ε and depending on the height of the stenosis, attains a very high asymptotic magnitude with increasing the catheter size, ε (Fig.3).

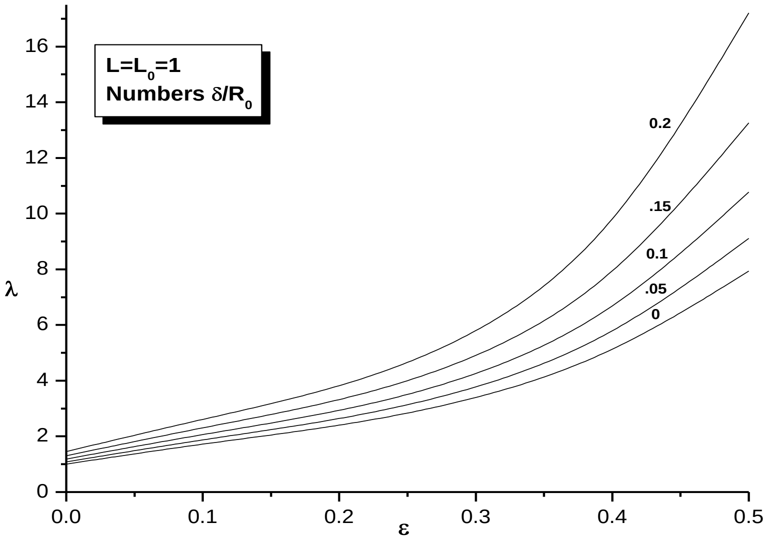

Fig.3 Impedance, λ versus catheter size, ε for different stenosis height, δ/R_0.

Table 2. Variations of impedance, λ with stenosis height, δ/R_0, and catheter size, ε

L_0=1						
δ/R_0	Impedance λ					
	L=1		L=2		L=5	
	$\varepsilon=0$	$\varepsilon=0.1$	$\varepsilon=0$	$\varepsilon=0.1$	$\varepsilon=0$	$\varepsilon=0.1$
0	1	1.74141	1	1.74141	1	1.74141
0.05	1.08261	1.89686	1.04131	1.81913	1.01652	1.7725
0.10	1.1825	2.0876	1.09125	1.91451	1.0365	1.81065
0.15	1.30501	2.32552	1.1525	2.03346	1.061	1.85823
0.20	1.45763	2.62782	1.22882	2.18461	1.09153	1.91869

Numerical results reveal that for any given set of other parameters, the impedance, λ decreases with increasing the tube length, 2L which interns implies that the flow resistance, λ increases with the stenosis length, $2L_0$ (Fig. 4) .

Figure 5-6 depict the wall shear stress in the stenotic region, τ_w with stenosis height, δ/R_0 and catheter size, ε. The wall shear stress in the stenotic region, τ_w increases rapidly in the upstream of the stenosis throat from its approached value at $z/L_0 = -1$ and achieves its maximal magnitude at stenosis throat (i.e., at $z/L_0 = 0$), it then decreases rapidly in the downstream of the throat to its approached value at the end point of the constriction profile (at $z/L_0 = 1$). The blood flow characteristic, τ_w increases with the stenosis height, δ/R_0 and the catheter size, ε at any axial location in the stenotic region (Fig. 5, 6).

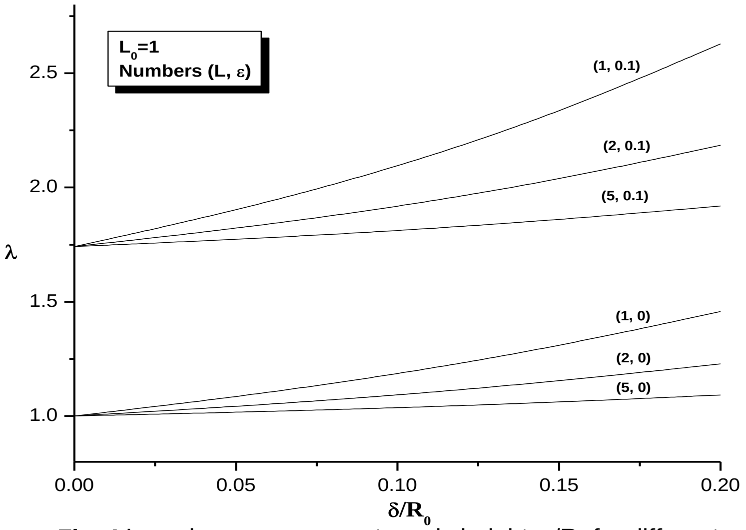

Fig. 4 Impedance, λ versus stenosis height, δ/R_0 for different length, L and catheter size, ε.

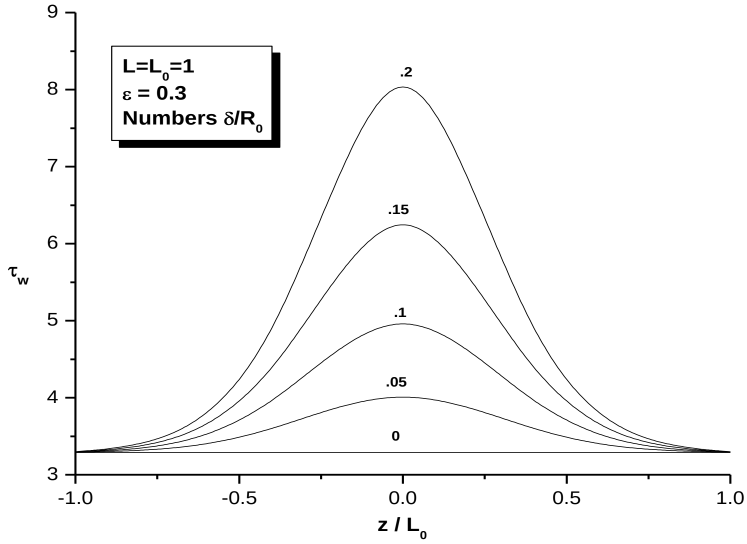

Fig.5 Wall shear stress distribution, τ_w in the stenotic region for different stenosis height, δ/R_0.

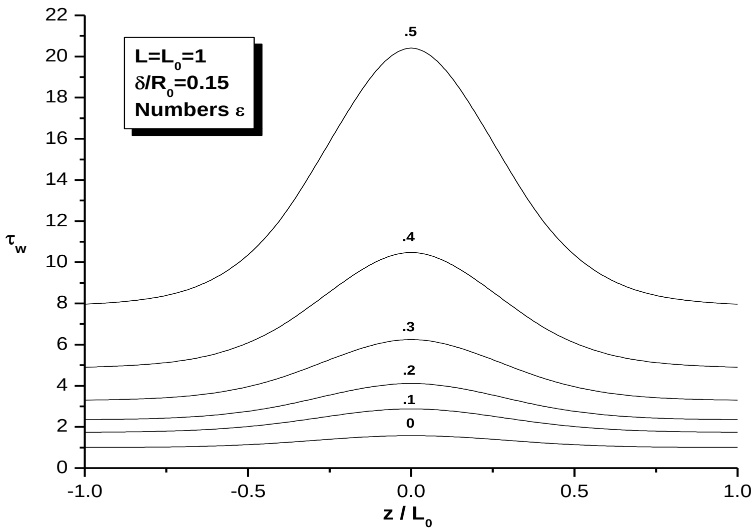

Fig.6 Wall shear stress distribution, τ_w in the stenotic region for different catheter size, ε.

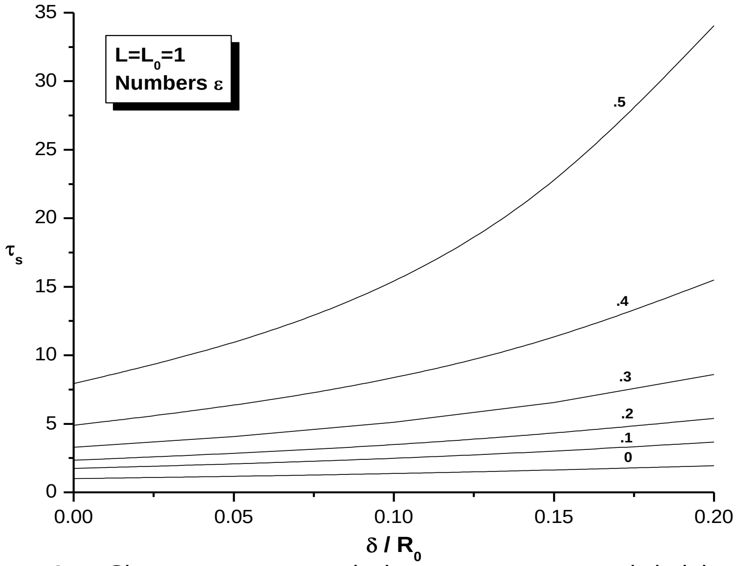

Fig.7 Shear stress at stenosis throat, τ_s versus stenosis height, δ/R_0 for different catheter size, ε.

Figure 7-8 depict the shear stress at the stenosis throat, with stenosis height, δ/R_0 and catheter size, ε. The shear stress at the stenosis throat, τ_s possesses the characteristics similar to that of the flow resistance, λ with respect to any parameter. However, the magnitude of the shear stress, τ_s is noted to be reasonably higher than the corresponding magnitude of the impedance, λ for any given set of parameters (Figs. 2, 3 and 7, 8).

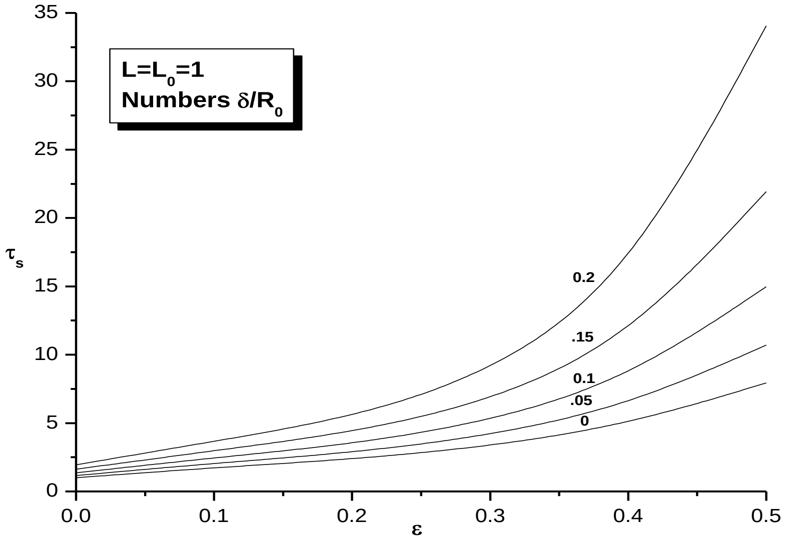

Fig.8 Shear stress at stenosis throat, τ_s versus catheter size ε for different, stenosis height, δ/R_0.

CONCLUSIONS

To estimate for the increased impedance and shear stress during artery catheterization, flow through a bell shaped stenosis has been analyzed assuming that the blood is represented by a Newtonian fluid. The blood flow characteristics (the flow resistance, the wall shear stress in the stenotic region and the shear stress at the stenosis throat) increase with catheter size as well as stenosis size (length and height). The shear stress at the stenosis throat possesses the characteristics similar to that of the impedance with respect to any parameter.

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