Scaling in Pressure Stimulated Currents related with rock fracture

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\textbf{A B S T R A C T}

The application of uniaxial stress in rocks is accompanied by the production of an electric signal which described by the term Pressure Stimulated Current (PSC). Here we present a statistical analysis of PSC time series from rock fracture experiments obtained from samples of calcite and cement mortar materials. In all considered cases, the PSC waiting time distribution can be described by a unique scaling function. This can be well approximated by a gamma function implying a broad distribution of waiting times. A frequency–energy distribution similar with the well known in seismology Gutenberg–Richter law and a Benioff-type relationship are obtained. The resulting scaling functions are similar to that for earthquakes and acoustic emissions suggesting its general validity for fracture processes.

\textbf{1. Introduction}

Despite the large amount of experimental data and the considerable effort undertaken \[1\], many questions about fracture have not yet been answered. Fracturing phenomena, particularly those concerning inhomogeneous materials, in association with transient electric phenomena, attract the interest of the scientific community. The main reason is that such phenomena are promising candidates of earthquake precursors \[2–4\]. It is well accepted that during the development of material deformation, there appear mechanisms that generate electric signal emission and a number of researchers acknowledge such mechanisms to be related to crack generation \[5–11\].

In order to understand the mechanisms that produce electric signals, a number of laboratory experiments have been conducted on dry and saturated minerals and rocks \[12–18\]. Furthermore, in scientific literature numerous studies and recordings of acoustic emissions due to mechanical stress caused microcracking in rocks have been reported \[19\]. The laboratory studies on electric signals emitted from rock specimens at the time of fracture, suggest a variety of mechanisms by which they are produced. These include the piezoelectric effect of quartz \[20\], the electrokinetic effect due to water filtration \[21\], point defects \[22\], emission of electrons \[23\]; and the motion of charged dislocations \[5,7–11\].

Recently, a series of laboratory experiments conducted on calcite samples have confirmed that the application of uniaxial stress is accompanied by the production of weak electric currents described by the term Pressure Stimulated Currents (PSC) \[9,18,24–29\]. Specifically, PSC and Acoustic Emission (AE) due to microcrack propagation precede macroscopic failure under conditions of constant stress or constant stress rate loading \[24–33\].

Pressure Stimulated Currents, acoustic emissions and earthquakes appear at different temporal and spatial scales and display considerable differences. In Ref. \[34\] it was shown that the Probability Density Function (PDF) of earthquake waiting
times — without distinguishing between foreshocks, main shocks, or aftershocks — for different areas, time windows and magnitude ranges can be described by a unique distribution, if time is rescaled with the mean rate of seismic occurrence. It was shown in particular, that the distribution holds from worldwide to local scales, for quite different tectonic environments and for all magnitude ranges considered. On the other hand, [35–37] indicate that the temporal, spatial and size distributions of AE events follow a power-law just as it is commonly observed in earthquakes.

In the present work we show that despite the aforementioned differences, the scaling law of the respective frequency-size distributions, the probability density function of the time interval between PSC events and a power-law behaviour of cumulative electric release rates, are similar to those observed in AE and seismicity. In particular, the PDF of pressure stimulated currents emitted during rock fracture experiments strongly suggest a universal character of the waiting time distribution and self-similarity over a wide range of activities in rock fracture. Furthermore, the fact that AE due to microcrack growth follows similar laws to the PSC, can be considered indicative that the motion of charged dislocations (i.e., the growth of microcracks), constitutes the driving mechanism of PSC as suggested in Refs. [7–11]. We analyse the PSC time series obtained from laboratory rock fracture experiments on calcite and cement samples, conducted in two modes: (a) constant stress and (b) at a constant stress rate.

2. Experimental procedure

Calcite is a geomaterial of known physical and chemical properties, which have been thoroughly presented in the previous works [24–29]. The samples under examination were collected from Mt. Pentели (Dionysos) and are mainly composed of calcite (98%) and other minerals, i.e. 0.5% muscovite, 0.3% sericite, 0.1% chlorite and 0.2% quartz [39]. Its specific density is 2730 kg/m³, apparent density 2717 kg/m³ and porosity is very low, varying from 0.3% for pristine calcite to 0.7% for calcite that has suffered natural weathering and has been exposed to corrosive agents [38,39].

Cement mortar samples were composed of Portland type cement (OPC - Ordinary Portland Cement), sand and water at a ratio of 1:3:0.5 respectively. The samples were used three months after their construction, so that they would age properly and achieve an approximate 95% of their maximum strength. The maximum diameter of the sand grains was 2 mm, their density was 2200 kg/m³ and porosity approximately 8%. Furthermore for the requirements of the measurements, a set of cement paste samples was prepared, with OPC and water ratio 1:0.5; drying time was 90 days.

Fig. 1 shows the experimental installation. The stressing system comprised a uniaxial hydraulic load machine (Enerpac–RC106) that applied the load to the samples. The stress was applied by a loading machine (model MTS–815) capable of applying a maximum force of ±1600 kN and a maximum deformation of ±50 mm. An integrated electronic micro-console (model MTS–453.20), equipped with a load and displacement controller, as well as a function generator unit were used to provide a closed loop control of the servo-hydraulic system. For the implementation of this experimental technique, a pair of gold plated copper electrodes were attached at the perpendicular axis to the stress. The measurements were made with a Keithley electrometer (model 6514). Electric measurements were stored in a computer hard disk through a GPIB interface, while the load cell and the strain gages bridge were guided to an A/D Keithley DAQ. The experiments were conducted in a Faraday shield to avoid electric noise. A detailed description of the loading and measuring systems, as well as of sample mounting and data recording, is given elsewhere [17,18]. Typical examples of time series recordings in calcite and cement are given in Figs. 2 and 3. Details on the experimental conditions are summarized in Table 1.

3. Scaling laws in pressure stimulated currents

In the following, we focus on two quantities that characterize each PSC event: (a) the square of PSC adjusted amplitude \( I_{psc} \), which expresses an energy quantity, i.e. \( E_i = \frac{I_{psc(i)}^2}{I_{max}^2} \), where \( I_{max} \) is the maximum value of the observed PSC, and (b) the time of occurrence of each observed PSC.
In Fig. 2(a)–(c) constant stress rate is applied to the sample. In Fig. 2(d) a very slow stress rate is applied as failure approached. In Fig. 3(a) a constant stress rate is applied and in Fig. 3(b) a step like stress followed by a constant load until failure was used. For clarity we present the failure stress $P_c$.

In the present experiments, the main source of PSC is the motion of charged dislocations [5,7–11]. It is therefore
inferred that a power-law behaviour may be fundamental, owing to the PSC being generated by a massive fracturing process, when spontaneously activated microcracks tend to coalesce leading to rock failure. Fig. 4 shows, not only that the influence of the specific material on the particular experiment and for any given fracture experiment, the PDF $P$ is band-limited. At both ends of the graph, there is a deviation from a Gutenberg–Richter type relationship. Similarly with earthquakes, the slope change effect at the high $\varepsilon$ range is explained as being due to the maximum energy release limited by the size of the stressed sample and by the energy density [40]. The shallow slope at the low $\varepsilon$ range might be related to either incomplete sampling of small events, ([30] and references therein), as with earthquakes and Acoustic Emission, or to a physical effect governing the process [41].

To this effect, the PSC series is transformed to a point process where events occur at times $t_i$ with $1 \leq i \leq N$, and therefore, the time between successive events can be obtained by $\tau_i = t_{i+1} - t_i$ [11,34]. These are the waiting times, also referred to as recurrence, or interevent times. The associated cumulative PDF is denoted by $P(\tau) = P(t_i > \tau)$. Fig. 5 shows the PDF of the normalized PSC waiting times $\chi = \tau/\langle \tau \rangle$ for calcite rocks and cement fracture experiments, where $\langle \tau \rangle = (t_N - t_1)/(N - 1)$ is the respective mean waiting time. The excellent data collapse implies that $P(\chi)$ does not depend on the particular experiment and for any given fracture experiment, the PDF $P(\chi)$ is determined by its mean waiting time $\langle \tau \rangle$ and the universal scaling function $P(\chi)$ which can be well approximated by a gamma distribution

$$P(\chi) \propto \chi^{-(1-\gamma)} \exp(-\chi/B)$$

with $\gamma \approx 0.8$ and $B \approx 0.7$. Therefore, we have essentially a decreasing power law with exponent about $1 - \gamma = 0.2$, up to the largest values of the argument, $\chi$ (close to 0.5), where the exponential factor comes into play. The latter expression is very similar to that observed for AE and earthquake data [30,35–37].

Figs. 4 and 5 show, not only that the influence of the specific material on $N(\varepsilon_i)/N_0$ and $P(\chi)$ is negligible, but also that the type of experiment (constant applied stress or constant stress rate) has no significant influence on them. Taking into account that PSC, AE and earthquake observations follow the same law, it is strongly suggested that $P(\chi)$ given in Eq. (1) is possibly a universal result for rock fracture. It further implies that $P(\chi)$ is self-similar over a wide range of activity rates, spanning at least 3 to 4 orders of magnitude for the experiments considered here.

Fracturing processes may be represented by the so-called “cumulative Benioff strain release” [42] $\Omega_\varepsilon(t)$, which relates the sum of the square root of the energy of emitted signals related to sequential fracture events, to the time prior to failure. Thus $\Omega_\varepsilon(t) = \sum \sqrt{\varepsilon_i}$, where the index $\varepsilon$ indicates application to electric signals like PSC. Hence, the “cumulative Benioff strain release” relation enables us to monitor the development of the scaling-up process. Furthermore, [42–45] and references therein showed that the analysis of cumulative Benioff strain release may provide an approximate magnitude and failure time for a major earthquake event. A proposed expression is $\Omega_\varepsilon = \Omega_{\text{sc}} - A_\varepsilon(t_{\varepsilon} - t)^{m_\varepsilon}$, where $t_{\varepsilon}$ is the failure time, while $A_\varepsilon$, $\Omega_{\text{sc}}$ and $m_\varepsilon$ are positive constants [43,44]. Since the energy parameter in the PSC recordings is $\varepsilon_i \propto I_{\text{psc}(i)}^2$, this can represent
a Benioff-type strain parameter, where
\[ \Omega_e(t) = \sum \sqrt{E_i} \propto \sum |\iota_{psc(i)}|. \]

Fig. 6a shows \( \Omega_{ec} - \Omega_e \) vs. \( t_c - t \). It is clear that as we approach failure, we obtain a linear relation with \( m_e \approx 0.8 \) to 0.9. The latter linearity is observed in Refs. [42–46] as well. We point out that the exponent \( m_e \) in general does not coincide with that observed for acoustic emission or seismological data. Considering a generation mechanism of PSC due to brittle damage evolution, the current density is
\[ j = \frac{\partial (\sum p_i)}{\partial t} = \frac{\partial \Pi(t)}{\partial t}, \]
where \( p_i \) and \( \Pi \) are the crack and the macroscopic polarization, respectively [7–11]. Taking into account that \( \Omega_e(t) \propto \sum |\iota_{psc}| \), after a simple calculation we conclude that \( \Omega_e(t) \propto \Pi(t) \). Considering irreversible thermodynamics for damage evolution and electric dipole generation as proposed in Ref. [47] and references therein, the Gibbs free energy per unit volume \( G \) is a function of entropy density \( s \), stress \( \sigma \), damage coefficient \( \alpha \) and polarization \( \Pi \). The total differential of \( G \) is given by the expression
\[ dG = -sdT - \frac{\partial G}{\partial \sigma} d\sigma - \frac{\partial G}{\partial \Pi} d\Pi + \frac{\partial G}{\partial \alpha} d\alpha, \]
where \( T \) is the absolute temperature, \( \frac{\partial G}{\partial \sigma} \) can be considered to be the macroscopic elastic strain \( \varepsilon \) and \( \frac{\partial G}{\partial \Pi} \) to be the electric field \( E \). As pointed out in Ref. [48], for an isothermal and linearly irreversible process, a combination of thermodynamic Maxwell relations with the damage coefficient evolution
\[ \frac{d\alpha}{dt} \propto (t_f - t)^{2-\rho \over \rho - 1}, \]
as suggested by damage mechanics [48], leads to
\[ \Pi(t) = \Pi_1 - \Pi_2 (t_f - t)^{1 \over \rho}, \]
where we note that \( \rho \) reflects the deformation mechanism of brittle and viscoelastic behaviour, with values usually varying between 2 and 6. Since \( \Omega_e(t) \propto \Pi(t) \) we conclude that \( \Omega_e \) has to follow a Benioff type time-to-failure power law with exponent \( m_e = 1/(\rho - 1) \). We clarify that as suggested in [47,48], acoustic emissions have to obey a similar Benioff type power law of time to failure, but with exponent \( m_{AE} = (\rho - 2)/(\rho - 1) \). For \( \rho \) close to 2, the value of \( m_e \) is slightly less than unity (e.g., for \( \rho = 2.25 \), \( m_e \approx 0.8 \)). When \( \rho = 4 \) then \( m_e = m_{AE} = 1/3 \), a value close to that observed in [42–49].

Furthermore, in Fig. 6b we present \( \Omega_e(t) \) vs. \( t_c - t \) on a bi-logarithmic scale, as was proposed in Ref. [49]. We point out that in the horizontal logarithmic axis, the time is reversed and we approach failure as we move from right to left. Thus, the first event appears at the right end of the diagram and failure at the left end. Fig. 6b as obtained using PSC recordings presents a remarkable similarity to the strain release results reported in Ref. [49] and facilitates the three principal phases of (a) nucleation, (b) intermediate and (c) irreversible to be distinguished.
4. Concluding remarks

To summarize, we have shown that Pressure Stimulated Currents obey a frequency–size law similar to the well-known Gutenberg–Richter law with a $b$-value close to 1. The probability density function for PSC waiting times is self-similar and can be described by a unique and universal scaling function $P(\chi)$. Its particular form can be well approximated by a gamma function implying a broad distribution of waiting times. This is very different from the exponential distribution expected for simple random Poisson processes and indicates the existence of a nontrivial universal mechanism in the PSC generation process. A power-law time-to-failure expression based on irreversible thermodynamics describes the cumulative energy.
release of PSC as failure approaches failure. All the aforementioned similarities with AE and seismicity and even the form of the power laws suggest a connection with fracture phenomena at much larger scales, implying that a basic general mechanism is “actively hidden” behind all these phenomena.

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References