ENERGY RESEARCH, VOL. 10, 181–188 (1986)

OPTIMIZATION OF A COMBINED WIND AND SOLAR POWER PLANT

J. C. HENNET

Laboratoire d'Automatique et d'Analyse des Systèmes, 7, avenue du Colonel Roche, 31077 Toulouse Cédex, France

AND

M. T. SAMARAKOU

Athens University, Electronics Laboratory, Solonos 104, Athens, Greece

SUMMARY

The paper is concerned with determining the optimized active areas of a photovoltaic conversion system, of a group of electricity generating wind machines and the optimal capacity of a battery storage system for a combined power plant. Minimization of the total life-cycle cost of the system is the criterion to obtaining the optimized parameters of the system. The algorithm consists of generating the system costs corresponding to various values of the parameters and to use these costs in a search procedure to determine the minimum. Each point is generated by a simulation program describing the system behaviour

KEY WORDS Wind power Solar power Energy system optimization

INTRODUCTION

The purpose of the work described in this paper is the optimization of a combined solar/wind power plant. Electricity is generated by photovoltaic panels, by wind generators and eventually by diesel sets.

This combined use of the renewable energy sources has been applied on the Greek island Kythnos. The existing system is equipped with a lead–acid battery storage system and a number of diesel generators. Previous work has described the characteristics of the system and the development of a simulation program (Samarakou and Hennet, 1985). The data obtained from the simulation indicated that an improvement of the system dimension could lead to decreasing the total life-cycle cost of the installation. The problems of frequency disturbances and of temporary unreliability of some devices are not considered in this paper because their effect on the system efficiency cannot systematically be attributed to one or another particular system component.

The proposed optimization technique is a pseudo-gradient type search procedure which allows the global minimal cost to be found after a limited number of iterations. Each simulation is run hour by hour over a period of one year. It is a simplified version of the Kythnos plant simulation, generalizing the application either to autonomous systems or to systems integrated with other energy sources.

PERFORMANCE EVALUATION OF THE SYSTEM

Performance evaluation of a combined system for a given set of dimension parameters is based on an hour by hour simulation. Numerical data used in the program are those of the Kythnos plant. The hourly energy balance is characterized by the net load NL_i and computed as follows:

$$NL_i = L_i - S_i Q - P_i R, \quad i = 1, \ldots, n$$

0363–907X/86/020181–08$05.00

Received 2 October 1984

with

L_i = electricity load during the ith period
S_i = electricity produced per unit of active area of the photovoltaic system during the ith period. It is computed from the hourly value of global radiation onto a horizontal surface, corrected by a factor depending on the tilt angle of the panels (37°25′). The conversion efficiency of the cells is supposed constant (with value 0·08).
Q = active area of the photovoltaic system
R = active area of the rotors of the wind machines
P_i = electricity produced per unit of active area of the wind machines during the ith period. Let us assume that the wind speed has the constant value V_i during the ith period.

The analytical expression for the wind machine output is

$$\begin{aligned} P_iR &= 0 &&, \text{if } V_i < V_{min} \\ P_iR &= \tfrac{1}{2}C_P\rho R V_i^3 &&, \text{if } V_{min} \leqslant V_i \leqslant V_r \\ P_iR &= P_r &&, \text{if } V_r \leqslant V_i \leqslant V_{max} \\ P_iR &= 0 &&, \text{if } V_i > V_{max} \end{aligned}$$

with

$$\begin{aligned} V_{min} &= \text{cut-in speed } (V_{min} = 3\,\text{m/s}) \\ V_r &= \text{rated speed } (V_r = 11{\cdot}10\,\text{m/s}) \\ V_{max} &= \text{cut-out speed } (V_{max} = 24\,\text{m/s}) \\ C_P &= \text{wind generator efficiency } (C_P = 0{\cdot}25) \\ \rho &= \text{air density } (\rho = 1{\cdot}3) \\ P_r &= \text{rated output } (P_r = \tfrac{1}{2}C_P\rho R V_r^3) \end{aligned}$$

We also denote by B_{min}, B_i, B_{max}, respectively, the minimal, the current and the maximal levels of electricity stored in the battery. The battery discharge efficiency is supposed constant with value $\eta (\eta = 0{\cdot}80)$.

Computation of NL_i leads to the different cases shown in Figure 1:

1. If $NL_i < 0$, there is an excess of production over electricity demand. We assume 100 per cent wind/solar penetration on the network, with the appropriate power and frequency regulation. All or part of the excess energy is then directed to the battery, depending on the comparison between $-NL_i$ and $(B_{max} - B_i)$.

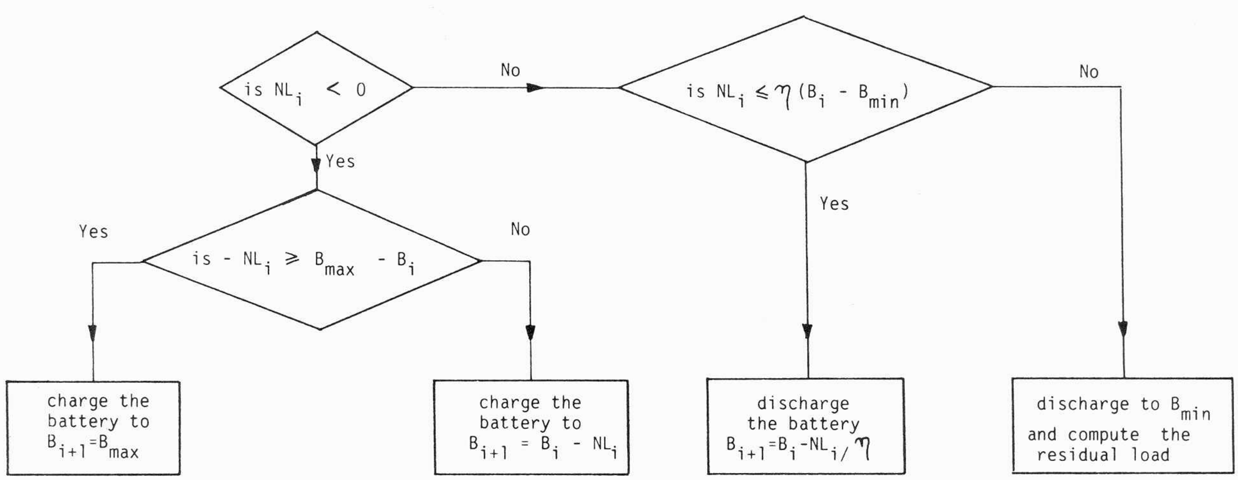

Figure 1. Possible operational cases

2. If $NL_i \geqslant 0$, the load is bigger than the electricity production from solar panels and wind machines. The operational configuration of the system depends on the value of X_i:

$$X_i = NL_i - \eta(B_i - B_{min})$$

(a) If $X_i > 0$, the net load cannot be totally met by wind and solar energies. The battery is discharged down to B_{min} and there is a residual load of value X_i.
(b) If $X_i \leqslant 0$, the load can be met and we get

$$B_{i+1} = B_i - NL_i/\eta$$

In case (a), nothing has been said about how to deal with the residual load. It actually depends whether or not the system is autonomous:

(i) If the system is coupled to other electricity generators, such as diesel units, the residual load X_i can be produced by these units at a cost which is a strictly increasing monotonic function of X_i for $X_i > 0$, $C(X_i)$, with $C(x) = 0$ for any $x < 0$.
(ii) If the system is autonomous, it may occasionally be unable to satisfy the demand and a failure may occur during one or several successive time periods. Such a breakdown causes a prejudice to the economic collectivity. The cost of the prejudice can be estimated as a function of the duration time and of the magnitude of the failure. The cost of the failure during the ith period is a strictly increasing monotonic function of X_i, for $X_i > 0$, also denoted $C(X_i)$ with $C(x) = 0$ for any $x \leqslant 0$. With this notation, we do not need to distinguish the two cases as long as the cost functions are not specified.

In order to evaluate the performance of the system, we use the hour as the basic time-period and the year as the evaluation period corresponding to the periodicity of the stochastic series $\{L_i\}$, $\{S_i\}$, $\{P_i\}$, $\{NL_i\}$ and $\{X_i\}$. We define the following quantities: $C_1(Q)$, $C_2(R)$, $C_3(B_{max})$; the prospective yearly costs of the solar system, of the wind machines and of the battery storage system as functions of their size. The yearly cost of each system includes the fixed costs (initial investments), and the variable costs (operation and maintenance). It is defined as the ratio of the present worth of the prospective cost cash flow at interest rate r by the life expectancy in years.

The evaluation function is the expected total yearly cost

$$Z = C_1(Q) + C_2(R) + C_3(B_{max}) + E\left[\sum_{i=1}^{n} C(X_i)\right]$$

with n the number of hours in one year ($n = 8760$).

Assuming that X_i is a random variable of probability density function $p_i(\cdot)$ during the ith period, we get

$$E[C(X_i)] = \int_0^\infty C(X)p_i(X)\,\mathrm{d}X$$

The stochastic series $\{X_i\}$ can theoretically be characterized from the statistical knowledge of the series L_i, S_i and P_i and from the value of the triplet (Q, R, B_{max}). Unfortunately calculation of $E\left[\sum_{i=1}^{n} C(X_i)\right]$ from the combined distributions of $\{L_i\}$, $\{S_i\}$ and $\{P_i\}$ is analytically intractable. Therefore, we prefer to generate three sample sequences which are the real data of the location selected for the plant. In the case of Kythnos, we use the real hourly data of the year 1982 for load values, for global solar radiation on a horizontal surface and for wind speeds.

Simulation of the plant over one year generates a sample sequence $\{X_i\}$ for each triplet (Q, R, B_{max}). The quantity $\sum_{i=1}^{n} C(X_i)$ can then be considered as an implicit function of the dimension parameters and denoted $C_4(Q, R, B_{max})$. The evaluation function actually used to compare the triplets (Q, R, B_{max}) is

$$Z = C_1(Q) + C_2(R) + C_3(B_{max}) + C_4(Q, R, B_{max})$$

OPTIMIZATION TECHNIQUE

We want to minimize the evaluation function Z subject to the operational constraints included in the simulation model. From the definition of functions C_1, C_2, C_3, C_4, we know that

$$\frac{\Delta C_1}{\Delta Q} > 0, \qquad \frac{\Delta C_2}{\Delta R} > 0, \qquad \frac{\Delta C_3}{\Delta B_{\max}} > 0$$

$$\frac{\Delta C_4}{\Delta Q} \leqslant 0, \qquad \frac{\Delta C_4}{\Delta R} \leqslant 0, \qquad \frac{\Delta C_4}{\Delta B_{\max}} \leqslant 0$$

at any point $(Q, R, B_{\max}) \in (R^+)^3$.

The combined system is designed for regions having enough solar and wind energies, so that there should exist a domain of values $(Q, R, B_{\max})$ for which

$$\frac{\Delta Z}{\Delta Q} = \frac{\Delta C_1}{\Delta Q} + \frac{\Delta C_4}{\Delta Q} \leqslant 0$$

$$\frac{\Delta Z}{\Delta R} = \frac{\Delta C_2}{\Delta R} + \frac{\Delta C_4}{\Delta R} \leqslant 0$$

$$\frac{\Delta Z}{\Delta B_{\max}} = \frac{\Delta C_3}{\Delta B_{\max}} + \frac{\Delta C_4}{\Delta B_{\max}} \leqslant 0$$

Since the values of $\Delta C_4/\Delta Q$, $\Delta C_4/\Delta R$, $\Delta C_4/\Delta B_{\max}$ keep increasing and tend to 0, when values of Q, R, $B_{\max}$ become larger, $\Delta Z/\Delta Q$, $\Delta Z/\Delta R$, $\Delta Z/\Delta B_{\max}$ can be supposed to be monotonically increasing functions of the variables Q, R, $B_{\max}$ at each point. Therefore we can conjecture that the Hessian matrix of the objective function is always positive definite. This means that the objective function can be assumed convex with a unique minimum value at the point $(Q^*, R^*, B^*_{\max})$ for which

$$\frac{\Delta Z}{\Delta Q}(Q^*, R^*, B^*_{\max}) = \frac{\Delta Z}{\Delta R}(Q^*, R^*, B^*_{\max}) = \frac{\Delta Z}{\Delta B_{\max}}(Q^*, R^*, B^*_{\max}) = 0$$

Under this assumption, we can use a gradient-type computational method. Since C_4 is an implicit function of $(Q, R, B_{\max})$, there is no analytical formulation of Z as a function of $(Q, R, B_{\max})$. Therefore we have to approximate derivatives by finite differences. The modified steepest descent algorithm (Polak, 1971) with an appropriate step length calculation (Armijo, 1966; Polak, 1971) generates the optimal solution of the problem.

The sequence of computational tasks to be performed is outlined as follows:

Notations : $X_j = (Q, R, B_{\max})_j$ at iteration j, $\Delta X = (\Delta Q, \Delta R, \Delta B_{\max})$

Data : $\alpha_1, \varepsilon_0, \beta_1, \alpha_2, \gamma, \beta_2, X_0$ with $\varepsilon_0 > 0$, $\beta_1 > 0$, $\gamma > 0$, $\alpha_2 > 0$, $\alpha_1 \in (0, 1)$, $\beta_2 \in (0, 1)$

Step 0 : Set $j = $, $\varepsilon = \varepsilon_0$

Step 1 : $h^j(\varepsilon, X_j) = -\frac{1}{\varepsilon}[Z(X_j + \varepsilon \Delta X) - Z(X_j)]$

Step 2 : Define $\Delta(\varepsilon, X_j) = Z[X_j + \beta_1 \varepsilon h^j(\varepsilon, X_j)] - Z(X_j)$

If $\Delta(\varepsilon, X_j) < 0$ go to step 3.

If $\Delta(\varepsilon, X_j) \geqslant 0$; set $\varepsilon = \varepsilon/2$ and go to step 1.

Step 3 : Determination of the step size λ_j

Step 3.1: set $\mu = \gamma$

Step 3.2: evaluate

$f(\mu, X_j, h^j) = Z[X_j - \mu h^j(\varepsilon, X_j)] - Z(X_j)$

and

$\theta(\mu, X_j) = f(\mu, X_j, h^j) + \mu \alpha_1 \| h^j(\varepsilon, X_j) \|^2$

Step 3.3:

if $\theta(\mu, X_j) \leqslant 0$, set $\lambda_j = \mu$

if $\theta(\mu, X_j) > 0$, set $\mu = \beta_2 \mu$ and go to step 3.2.

Step 4 : If $f(\lambda_j, X_j, h^j) \leqslant -\alpha_2\varepsilon$, set $X_{j+1} = X_j + \lambda_j h^j(\varepsilon, X_j)$, set $j = j+1$ and go to step 1 for the next iteration.
If $f(\lambda_j, X_j, h^j) > -\alpha_2\varepsilon$, $X^* = X_j$, stop.

Theoretically, the convergence of this algorithm is guaranteed and the practical implementation has shown good convergence rates.

We used the following set of parameter values:

$$\alpha_1 = 0{\cdot}5,\ \alpha_2 = 0{\cdot}05,\ \varepsilon_0 = 10,\ \beta_1 = 5,\ \beta_2 = 0{\cdot}5,\ \gamma = 3$$

The example of Figure 2 has been obtained for Kythnos data with the initial dimension values of the real plant:

$$X_0 = (1200\ \mathrm{m}^2,\ 528{\cdot}5\ \mathrm{m}^2,\ 600\ \mathrm{kWh})$$

RESULTS

In spite of the fact that Kythnos data show a relatively high level of yearly insolation, the optimization results relative to this site indicate that the most profitable solution does not include any solar component. It consists of a large wind power plant with great support from the battery storage system. This is due to the large average yearly cost of the photovoltaic conversion system, estimated at 50\$/m^2.

We developed a prospective scenario with mass production of photovoltaic cells at a lower cost. We also assumed either the case of an autonomous system or the case of an expensive auxiliary energy source, which both led to increasing the slope of the cost function $C(\cdot)$. We assumed that average yearly cost functions are linear:

$$C_1(Q) = C_1 Q \text{ with } C_1 = 31\$ \text{ per m}^2 \text{ of photovoltaic panels}$$
$$C_2(R) = C_2 R \text{ with } C_2 = 55\$ \text{ per m}^2 \text{ of rotor swept area}$$
$$C_3(B_{\max}) = C_3 B_{\max} \text{ with } C_3 = 12\$ \text{ per kWh of installed capacity}$$
$$C(X) = CX \text{ with } C = 0{\cdot}3\$ \text{ per kWh of residual load.}$$

Then, the optimization gave the following results:

$$Q = 300\ \mathrm{m}^2,\ R = 1160\ \mathrm{m}^2,\ B = 880\ \mathrm{kWh}$$

In order to check the convexity assumption on which we based the construction of the optimization technique, we proceeded to a sensitivity analysis around the optimal value for each of the three parameters. As shown in Figures 3–5, the convexity assumption is verified. The local minimum found by the computational

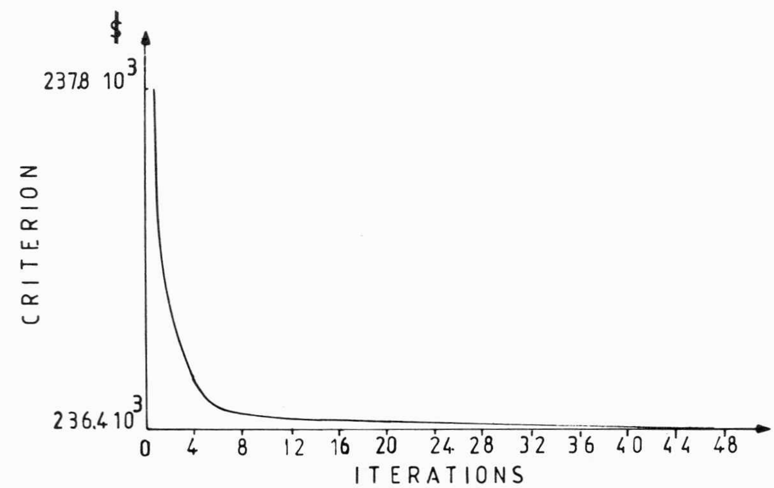

Figure 2. Evolution of the computational procedure

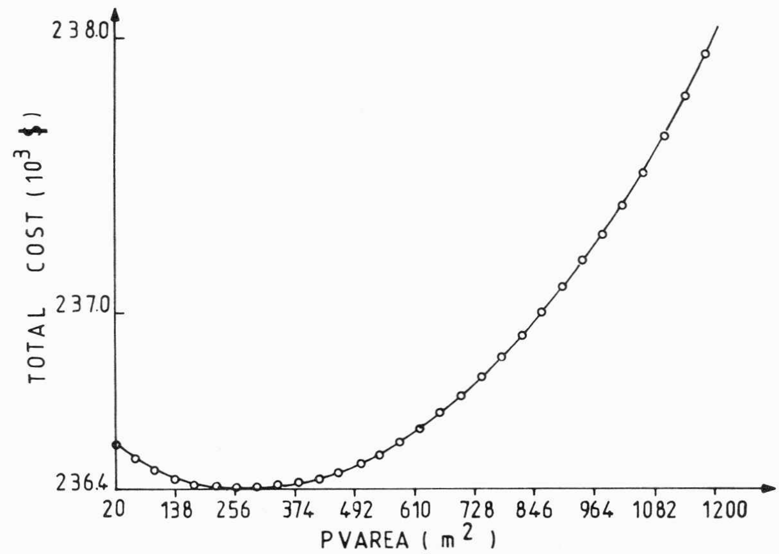

Figure 3. Variations of Z with Q around the optimum

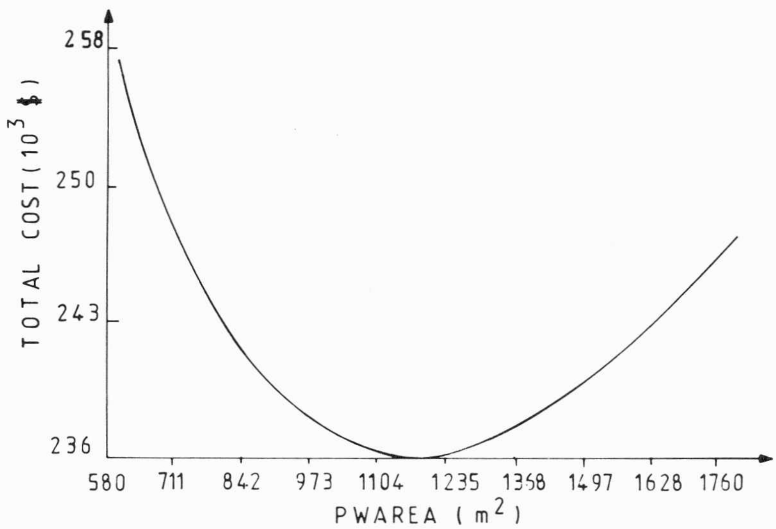

Figure 4. Variations of Z with R around the optimum

technique is the global minimum of the problem. Figure 6 shows a comparison of the sensitivity of the total cost to variations of the three subsystems dimensions around their optimal value.

CONCLUSION

Several studies have shown the interest of combining electricity generation by a photovoltaic plant and by a wind energy system. The complementarity of these two renewable energies can be improved through battery storage units connected to both systems, which helps adjusting the electricity production to the load.

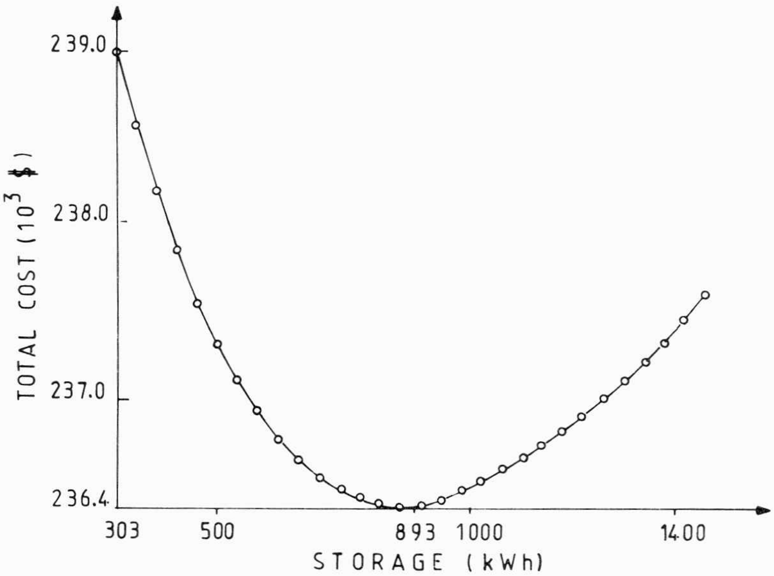

Figure 5. Variations of Z with B_{max} around the optimum

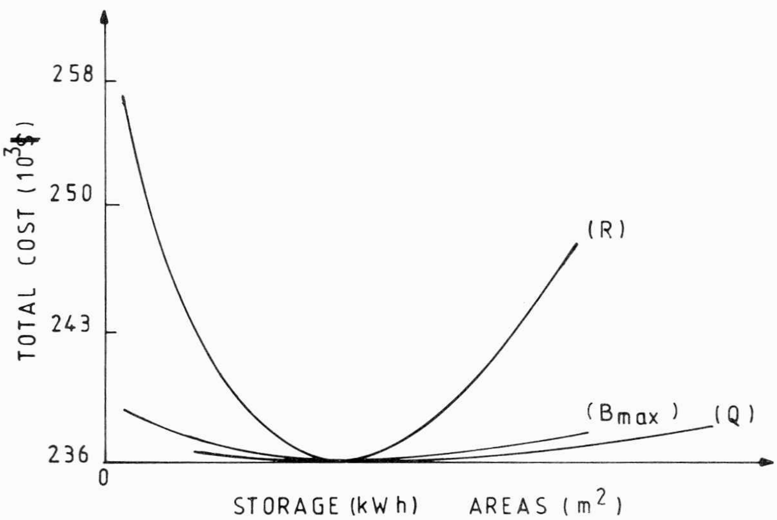

Figure 6. Partial variations of Z around the optimum

The problem of determining the optimal sizes of the three components has been formulated for an autonomous system and for an integrated system with an auxiliary electricity generator such as diesel units. In spite of the implicit nature of the criterion as a function of the sizes, the computational technique is efficient and easy to use. It can be applied to any set of local data as a help in conceiving a combined plant.

The numerical values obtained in this paper are relative to the example of Kythnos for which the meteorological and the load data were available. They should only be considered as an illustrative example since the evaluated cost values could not be accurately determined. The difficulty of *a priori* cost/benefit analysis arises from the facts that the photovoltaic equipment has not yet been mass produced and that the two renewable energy production systems are still at an experimental level. But the economic analysis is nevertheless a convenient way to the design of a coherent combined system.

REFERENCES

Armijo, L. 1966. 'Minimization of functions having continuous partial derivatives', *Pacific J. Math*, **16,** 1–3.
Infeld, D. G., Slack, G. W., Lipman, N. H. and Musgrove, P. J. 1983. 'Review of wind/diesel strategies', *IEE Proc*, **130,** pt A, (9), (1983).
Joubert, A. and Pecheux, J. 1981. 'Etude du comportement d'un système énergétique fonctionnant à partir des énergies solaire et éolienne', *Revue de Physique Appliquée*, **16,** (7), (1981).
Polak, E. 1971. *Computational Methods in Optimization* (*Mathematics in Science and Engineering, vol.* 77), Academic Press.
Samarakou, M. T. and Hennet, J. C. 1986. 'Simulation of a combined wind and solar power plant', *Int. J. Energy Res.* **10,** 1–10.