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On the thermal time constant of structural walls

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Abstract

In the present investigation a numerical model is employed for the calculation of the transient wall heat flux, under the effect of a step temperature change at a particular wall side, for given initial and boundary conditions. The results were employed for the evaluation of the wall thermal time constant, which is widely employed quantity in Physics and technology to describe the transient output response characteristics of a system under the influence of a step change of a particular input variable parameter.

According to the derived results, it was found that the wall thermal time constant is a wall side specific quantity, something which leads to the definition of two time constants, the forward and reverse, which conventionally correspond to the room and ambient wall sides respectively. Furthermore, their ratio appears also to be a very interesting quantity, which represents the measure of the uneven distribution of heat capacity in respect of the wall plane of symmetry and becomes unity for symmetrical walls. Both thermal time constants were evaluated for a number of typical walls, broadly employed in construction practice, and the results were comparably presented and discussed in respect to their fundamental significance on the load characteristics and energy demand of buildings.

Keywords: Transient wall thermal behaviour; Wall heat conduction; Wall transient response; Wall thermal time constant

1. Introduction

Walls are very important structural elements, which are designed with mechanical and thermal criteria of buildings in mind. Although the first group of criteria are imposed by the layout and function of a construction, the climatic control design considerations are mainly imposed by

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Nomenclature

A	wall surface area (m^2)
c	specific heat capacity (J/kg K)
E	energy (J)
h	heat transfer coefficient (W/m^2 K)
HC	wall heat capacity (J/K)
k	thermal conductivity (W/m K)
L	wall thickness (m)
M	wall mass (kg)
q	heat flux rate (W/m^2)
R	thermal resistance (K/W)
t	time (s)
T	temperature (K or °C)
T^*	time period (s)
TC	wall thermal time constant (s)
U	wall "U" value (W/m^2 K)
x	space coordinate (m)
Greek letters	
δ	wall sublayer thickness (m)
ΔT	temperature difference (°C)
Δx	space step (m)
ρ	density (kg/m^3)
Subscripts	
r	room side
eq	equivalent
F	forward
i	initial
o	amplitude, step change
R	reverse
s	steady-state
∞	ambient side

environmental parameters. Apart from contributing to the significant energy conservation during the entire life span of the buildings by controlling the energy exchange between space and environment, they also promote the development of a natural comfortable building environment. This heat exchange is determined by the wall thermal characteristics, which should be selected appropriately in order to control the local climatic conditions. Although the wall thermal behaviour is usually determined through both thermal resistance and heat capacity, a physical quantity of a particular importance for the classification of the structural wall thermal behaviour is the wall thermal time constant, which is broadly employed in Physics to describe the delay in the output response of a system under the effect of an instantaneous variation of an input parameter.

Assume an isothermal plane wall at a uniform initial temperature T_i, which exchanges heat with the ambient and room air at both sides by convection. When ambient and room air temperatures are both fixed at T_i, the system is at a thermal equilibrium with its environment so as no heat exchange occurs through both wall surfaces. Any step temperature change of room or ambient air at $t = t_o$, causes the development of a corresponding transient heat flux $q(t)$, which gradually converges to its steady-state value q_s. Wall thermal time constant can be defined as the time required for the wall heat flux to attain a state of 36.78% of the difference between the initial at $t = t_o$ and the steady-state heat flux values, $q(t_o)$ and q_s, respectively.

Although the pure mathematical problem has been treated in extensive detail in classical texts [1], several other analytical approaches have so far appeared in the literature [2–5]. Numerical analysis however, and more specifically the finite difference method, has became the most powerful method which is extensively employed for the development of approximate solutions, especially for analyses involving composite walls and complex boundary conditions as they are easily adaptive to contemporary computers [6–8]. The numerical solution of the one-dimensional conduction heat transfer equation is derived for a broad range of frequently applied wall designs, which is employed for the evaluation of their thermal time constants.

2. Theoretical analysis

The conduction heat transfer in the mass of a plane wall of large lateral dimensions is described by the one-dimensional transient heat conduction equation

$$\frac{\partial}{\partial x}[k \cdot (\partial T/\partial x)] = \rho \cdot c \cdot (\partial T/\partial t) \tag{1}$$

where k, ρ and c the thermal conductivity, density and specific heat capacity of the wall material which are generally functions of the coordinate x.

Assuming that the coordinate system is located at the surface of the wall, it would be possible to deliberately define as ambient and room side of the boundary wall surfaces at $x = 0$ and $x = L$ respectively. It is also assumed that the wall exchanges heat by convection with both the room and ambient air at the temperatures of $T_r(t)$ and $T_\infty(t)$ respectively. Then, the boundary conditions of the problem are expressed in terms of the temperature gradient and convective heat fluxes at both wall surfaces,

$$q(0,t) = -k \cdot (\partial T/\partial x)|_{x=0} = h_\infty \cdot [T_\infty(t) - T(0,t)] \tag{2}$$

and

$$q(L,t) = -k \cdot (\partial T/\partial x)|_{x=L} = h_r \cdot [T(L,t) - T_r(t)] \tag{3}$$

where h_∞ and h_r are the convective heat transfer coefficients for both the ambient and room sides respectively. As initial condition it is assumed that the wall is isothermal at the temperature T_i, at $t < t_o$. The room side of the wall is subjected to a step air temperature change at $t = t_o$, so as

$$T_\infty(t) = T_r(t) = T_i, \quad \text{at } t < t_o \tag{4}$$

and

$$T_r(t) = T_o, \quad \text{at } t \geqslant t_o \tag{5}$$

This leads to the development of transient heat fluxes, which gradually converge to their steady-state values when all transients have died away. As soon as the initial and the two boundary conditions (2) and (3) are specified, solutions of Eq. (1) can easily be derived employing the finite difference method.

The composite wall was subdivided into sublayer sections of uniform temperature and thermophysical properties, at the centre of which a node of a thermal network was located. Eq. (1) with the boundary conditions (2) and (3) was translated into a system of simultaneous algebraic equations for each inner and boundary node, using the implicit, inherently stable Laasonen finite difference scheme, while the selection of the optimum number of sublayer sections and time steps was based on numerical experiments as discussed in greater detail in [8]. According to the first preliminary simulations it was found, that for the range of the investigated wall designs, the transient wall response to a step room air temperature change lasts about 50 h prior to the asymptotic approach to steady-state solutions. For this reason and for a typical sublayer section thicknesses and time steps of 1 cm and 120 s respectively, a time domain of 96 h was found to be adequate for all simulations.

3. A classification of wall designs

The thermal behaviour of ordinary structures is mainly determined by the specific design of the building shell, the vertical elements of which are usually bearing walls or combination of curtain walls with skeleton load-supporting frames. Bearing walls are usually made of simple reinforced concrete slabs. They may often also be heavier and more complex design structures of thermally insulated reinforced concrete slabs, layered at both sides by solid bricks or plaster, or scarcely very heavy masonry walls. Since the majority of contemporary buildings are made with skeleton frames, the curtain walls which are designed to support no load other than their own weight, are becoming very important structural elements for the thermal behaviour of buildings. The wall thickness which determines its heat capacity is mainly imposed by the mechanical load although fire-resistance requirements are sometimes also considered. In certain geographical areas with less stringent regulations and lack of seismic loads, the walls can be of a more simple design, composed mainly of hollow concrete blocks and bricks with dead air space. For certain category of structures like industrial buildings, walls they can simply be made by a solid foamed thermal insulation layer, enclosed between light-gage metal sheets. When the thermal insulation slab is layered at both sides by plaster, wood or gypsum, it is usually referred to as lightweight panel wall.

Although the range of walls employed in contemporary structures is very wide, only a small number of characteristic designs was selected for consideration in the present investigation, based mainly on descriptions, recommendations and general guidelines from the relevant literature [9–11].

A brief description of the investigated wall designs is shown in Fig. 1 in which the overall wall thickness L is also indicated. Individual wall layer thicknesses δ_1, δ_2, δ_3, δ_4 and δ_5 are shown in

Table 1 in which the material designation and the thermophysical properties k, ρ and c are also shown.

Very simple walls of a fundamental design are the lightweight panels, consisting typically of wood or fiber board and gypsum layers hold in place between wood studs. These walls with rigid or blanket thermal insulation layers installed in the so formed air gaps between wood studs, are shown as walls 1 and 2 corresponding to thermal insulation layers of 4 and 10 cm thick respectively. Designs 3–7 are heavier walls consisting mainly of brick layers. The walls 3 and 6 are walls consisting of thick face brick layers at the ambient side, followed by either a thermal insulation or a common hollow brick layer respectively. The walls 4 and 5 are common symmetrical double row hollow brick walls, the latter being insulated by a thermal insulation layer placed in the air gap between the brick rows. Wall 7 consists of a face brick in contact with a thermal insulation layer, followed by a hollow concrete block layer. Finally the designs 8–10 represent bearing walls consisting of thick reinforced concrete layers. At the room side of the wall 8, the thermal insulation is placed next to the reinforced concrete layer, the system being plastered at both sides, while in the wall 10 which is otherwise identical to the wall 8 the thermal insulation layer is

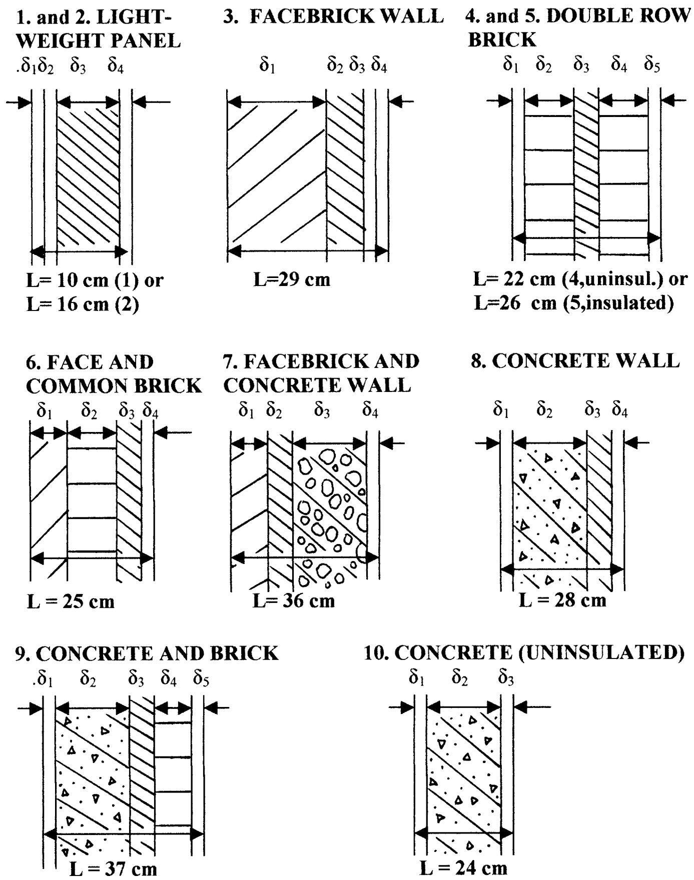

Fig. 1. The design description of the walls 1–10.

Table 1
Wall layer thicknesses, thermophysical properties and calculated U and $(\rho \cdot c)_{eq}$ values of the walls 1–10

Wall description and type	Wall layer	Material description	Thickness (cm)	k (W/m K)	ρ (kg/m^3)	c (J/kg K)	U (W/m^2 K)	$(\rho \cdot c)_{eq}$ (J/kg K)
Lightweight panel (walls 1 and 2)	δ_1	Siding wood	2	0.20	460	1400	0.7473 (0.3569)	690.4 (444.2)
	δ_2	Fiber board	2	0.28	500	1300		
	δ_3	Thermal insulation	4 (10)	0.041	40	850		
	δ_4	Gypsum board	2	1.25	1900	1100		
Facebrick wall (wall 3)	δ_1	Face brick	20	1.20	1900	850	0.6069	1301.2
	δ_2	Thermal insulation	4	0.041	40	850		
	δ_3	Vegetable fiber board	3	0.095	400	800		
	δ_4	Plaster	2	1.39	2000	1085		
Double row brick (walls 4 and 5)	δ_1	Plaster	2	1.39	2000	1085	1.9853 (0.6760)	1402.5 (1192.0)
	δ_2	Common brick	9	0.60	1400	880		
	δ_3	Thermal insulation	0 (4)	0.041	40	850		
	δ_4	Common brick	9	0.60	1400	880		
	δ_5	Plaster	2	1.39	2000	1085		
Face and common brick (wall 6)	δ_1	Face brick	10	1.20	1900	850	0.7151	1268.5
	δ_2	Common brick	9	0.60	1400	880		
	δ_3	Thermal insulation	4	0.041	40	850		
	δ_4	Plaster	2	1.39	2000	1085		
Facebrick–concrete wall (wall 7)	δ_1	Face brick	10	1.20	1900	850	0.6740	1595.1
	δ_2	Thermal insulation	4	0.041	40	850		
	δ_3	Hollow concrete block	20	0.85	2000	920		
	δ_4	Plaster	2	1.39	2000	1085		
Insulated concrete wall (wall 8)	δ_1	Plaster	2	1.39	2000	1085	0.7710	1605.5
	δ_2	Concrete layer	20	1.70	2300	920		
	δ_3	Thermal insulation	4	0.041	40	850		
	δ_4	Plaster	2	1.39	2000	1085		

Table 1 (*continued*)

Wall description and type	Wall layer	Material description	Thickness (cm)	k (W/m K)	ρ (kg/m^3)	c (J/kg K)	U (W/m^2 K)	$(\rho \cdot c)_{eq}$ (J/kg K)
Concrete and brick wall (wall 9)	δ_1	Plaster	2	1.39	2000	1085	0.6910	1632.0
	δ_2	Concrete layer	20	1.70	2300	920		
	δ_3	Thermal insulation	4	0.041	40	850		
	δ_4	Common brick	9	0.60	1400	880		
	δ_5	Plaster	2	1.39	2000	1085		
Uninsulated concrete wall (wall 10)	δ_1	Plaster	2	1.39	2000	1085	3.1113	1867.5
	δ_2	Concrete layer	20	1.70	2300	920		
	δ_3	Plaster	2	1.39	2000	1085		

completely lacking. The wall 9 is an even heavier design version of the concrete walls, including an additional hollow brick layer next to the thermal insulation layer at the room side.

The U value and the equivalent heat capacity are very simple characteristic wall quantities. The U value of a composite wall made of n layers of uniform thermophysical properties, surface area A and thermal resistance R is defined as $U = (A \cdot R)^{-1}$ and is calculated by the expression,

$$U = \left[1/h_{\mathrm{r}} + \sum_{i=1}^{n} (\delta_i / k_i) + 1/h_{\infty} \right]^{-1} \tag{6}$$

where δ_i and k_i are the layer thicknesses and thermal conductivities as shown in Fig. 1 and Table 1 respectively, while its value can also be evaluated by standard laboratory procedures [12].

The physical quantity which represents the heat storage capability is the wall heat capacity, defined as $\mathrm{HC} = M \cdot c$. While the per unit wall area of this quantity is $(\mathrm{HC}/A) = \rho \cdot c \cdot \delta$, where δ the wall thickness, the per unit volume wall heat capacity, being a characteristic wall quantity independent from the wall thickness, is equal to $\rho \cdot c$. This quantity for a composite wall of an overall thickness L, is usually defined as the equivalent per unit wall volume heat capacity and it is expressed as,

$$(\rho \cdot c)_{\mathrm{eq}} = (1/L) \cdot \sum_{i=1}^{n} (\rho_i \cdot c_i \cdot \delta_i) \tag{7}$$

where ρ_i and c_i are the densities and the specific heat capacities of the n parallel layers of the composite wall.

For the wall designs 1–10 in Fig. 1 and for the selected values of thermophysical properties, the calculated quantities U and $(\rho \cdot c)_{\mathrm{eq}}$ are shown in Table 1. It is remarkable to notice the low U values and per unit volume heat capacities of the lightweight panel wall types 1–2, as compared to the heavier concrete walls.

4. Results and discussion

For the purpose of the present investigation, it is assumed that the walls are initially isothermal at the uniform temperature of $T_i = 5$ °C. At the time $t = t_o$, the room side of the wall is exposed to a fixed air temperature of $T_r = 20$ °C, which is equivalent to a positive step room air temperature change of 15 °C, while the convective heat transfer coefficients at ambient and room sides are $h_\infty = 20.0$ W/m^2 K and $h_r = 8.0$ W/m^2 K respectively.

Owing to the large temperature differences, the developed time-depended convective heat flux between the wall and the surrounding room air being maximum at $t = t_o$, decreases exponentially with time and approaches asymptotically its steady-state value. The time-depended heat flux is calculated by the expression,

$$q(L,t) = h_r \cdot [T_r - T(L,t)] \tag{8}$$

where $T(L,t)$ is the wall temperature. Following the definition and assuming that the maximum transient and steady-state indoor wall heat fluxes are $q(L,t_o)$ and q_s respectively, the time-depended value of $q(L,t) - q_s$ is given by the following exponential expression,

$$\frac{q(L,t) - q_s}{q(L,t_o) - q_s} = \exp(-t/\mathrm{TC}) \tag{9}$$

where TC is the wall thermal time constant. For $t = \mathrm{TC}$, it is derived that

$$q(L,\mathrm{TC}) = q_s + \exp(-1) \cdot [q(L,t_o) - q_s] \tag{10}$$

which means that at $t = \mathrm{TC}$, the transient heat flux $q(L,\mathrm{TC})$ is the sum of the steady-state plus the 36.78% of the difference between maximum and steady-state heat flux. When the solution $q = q(L,t)$ of the differential equation (1) is derived, one can easily determine the wall time constant $t = \mathrm{TC}$, directly from Eq. (10).

Referring to the walls under consideration, it was found that the calculated time constant is a physical quantity which is associated to the specific wall side to which the specified temperature change is applied. This necessarily leads to the consideration of two time constants, the first of which can be referred as the forward and the second as the reverse wall thermal time constant. It is deliberately assumed that the forward time constant is associated to the room side, while the reverse to the ambient side of the wall. Following this convention, both the forward and reverse time constants are irrelevant to the direction of the heat flow. For example, the forward time constant may be referred either to a negative (heat loss) or a positive (heat gain) direction of heat flow, as soon as the temperature step change of the room air is positive or negative respectively. The same is valid for the reverse time constant. The definition of the two time constants, which are generally not equal, is attributed to the uneven distribution of mass and heat capacity in respect to the wall plane of symmetry. While unity ratio between both time constants means full symmetry of wall geometry and materials, the deviation from unity ratio value is a measure of uneven distribution of heat capacity in respect of the wall plane of symmetry.

The numerical solution of (1) under the appropriate initial and boundary conditions leads to high transient heat fluxes at $t = t_o$, which are exponentially decreasing to their corresponding steady-state values when all transients have died away. This was found to happen within the specified time domain of 96 simulation hours. It was also found that the so derived steady-state heat fluxes are always practically identical to the corresponding values derived by the simple steady-state considerations,

$$q_{\mathrm{s}} = \Delta T \bigg/ \left[1/h_{\mathrm{r}} + \sum_{i=1}^{n} (\delta_i/k_i) + 1/h_{\infty} \right] \tag{11}$$

As soon as the forward thermal time constant is calculated, the step temperature change and the corresponding heat transfer coefficients are replaced from the room to the ambient side of the wall and the same calculations are carried out, revealing the reverse wall thermal time constant.

The derived solutions $q = q(L, t)$ of Eqs. (1)–(3) for the walls 1–10 were plotted and the results are presented in Figs. 2–4. The negative sign in the calculated heat flux signifies the direction of heat flow, which owing to the employed positive temperature steps is always considered as a heat loss.

In Fig. 2 the developed forward transient heat fluxes for the lightweight panel and the facebrick and concrete walls 1, 2 and 7 respectively were plotted in solid lines, while the corresponding values for the reverse heat fluxes are shown in dotted lines. It can be seen that following a relatively short transient of about 7 h during which as high as 120 W/m^2 heat fluxes occur, steady-state solutions are developed which are lower for the wall 1, owing to the thicker thermal insulation layer employed. When the ambient wall side is subjected to a step change of air temperature, the transient is even shorter, owing to the lack of symmetry in the mass distribution of the wall layers. As can be seen from the same plot, although the reverse transient for the wall 7 lasts about 36 h, the forward being about 75 h is appreciably longer, owing to the presence of a thick hollow concrete block layer close to the room side of the wall. In Fig. 3 the corresponding behaviour for the walls 3 and 4 and 8 is shown. It can clearly be seen, that for the walls 3 and 8 the forward transient of about 10 h is appreciably shorter than the corresponding reverse of about 60 and 77 h respectively. This is attributed to the comparatively small heat capacity of the wall layers

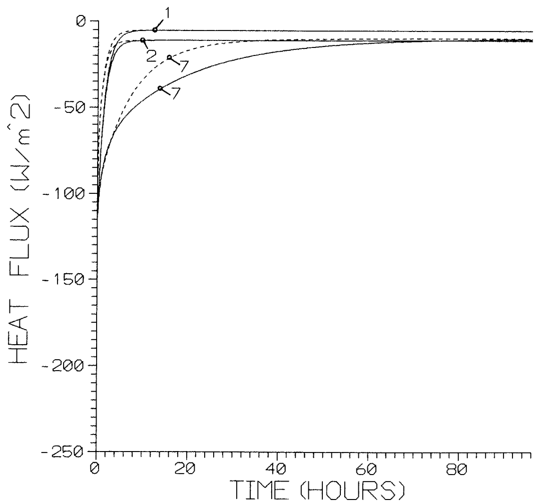

Fig. 2. The calculated time-depended heat flux at the wall boundaries, under the influence of a step temperature change of air at the room (solid line) or the ambient (broken line) side, for the walls 1, 2 and 7.

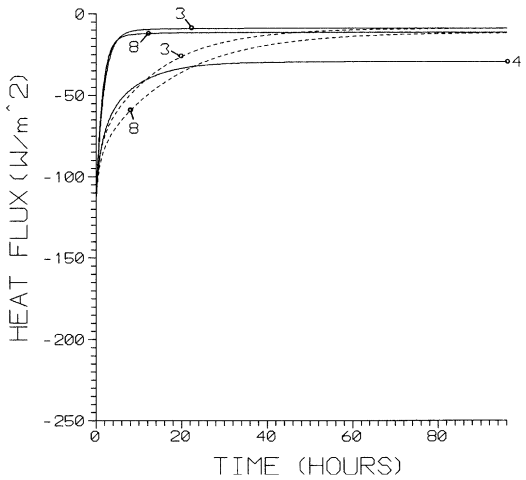

Fig. 3. The calculated time-depended heat flux at the wall boundaries, under the influence of a step temperature change of air at the room (solid line) or the ambient (broken line) side, for the walls 3, 4 and 8. Solid and broken lines for the wall 4 are identical owing to the full symmetry of wall layers.

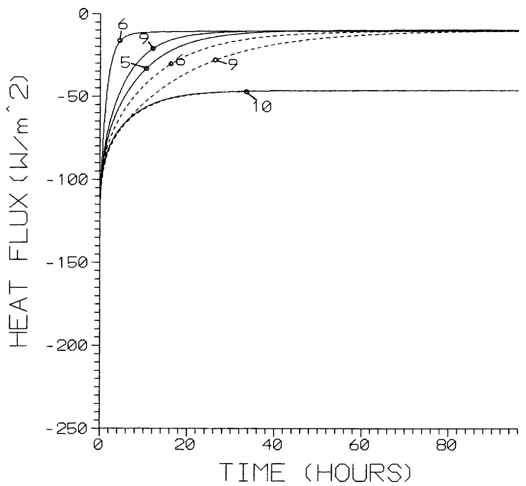

Fig. 4. The calculated time-depended heat flux at the wall boundaries, under the influence of a step temperature change of air at the room (solid line) or the ambient (broken line) side, for the walls 5, 6, 9 and 10. Solid and broken lines for the walls 5 and 10 are identical owing to the full symmetry of wall layers.

close to room as compared to the thick face brick or concrete layers for the walls 3 and 8 respectively, at the ambient side. This leads to a substantial difference between forward and reverse wall time constants as shown in Table 2. Owing to the complete symmetry the forward and reverse heat fluxes for the wall 4 are identical.

In Fig. 4, the corresponding results are plotted for the walls 5, 6, 9 and 10. Again, owing to the full symmetry, the forward and reverse heat flux curves for the walls 5 and 10 are identical. Besides, the shape of the forward and reverse heat fluxes for the wall 6 is completely different, leading

Table 2
Calculated values of forward $(TC)_F$ and reverse $(TC)_R$ time constants and transient duration times T_F^* and T_R^* respectively for the walls 1–10

Wall type	Wall description	Forward time constant $(TC)_F$ (h)	Reverse time constant $(TC)_R$ (h)	Transient duration T_F^* (h)	Transient duration T_R^* (h)
1	Lightweight panel ($\delta_3 = 4$ cm)	1.600	1.102	7.100	6.125
2	Lightweight panel ($\delta_3 = 10$ cm)	1.480	0.960	6.710	5.300
3	Facebrick wall	1.790	9.250	9.790	60.33
4	Double row brick (uninsulated)	3.960	3.960	32.67	32.67
5	Double row brick (insulated)	6.290	6.290	39.30	39.30
6	Face and common brick	1.562	8.660	9.330	53.55
7	Face brick and concrete block	9.220	6.480	75.38	36.12
8	Concrete wall (insulated)	1.530	19.42	9.50	77.28
9	Concrete and brick wall	4.560	13.02	28.50	79.44
10	Concrete wall (uninsulated)	4.353	4.353	30.08	30.08

to an appreciably shorter forward transient of about 9 h as compared to 54 h for the reverse. This is attributed to the strongly uneven distribution of the wall heat capacity, which is caused by the presence of a thick face brick layer close to the ambient side of the wall.

The corresponding behaviour for the wall 9 indicates a substantially longer reverse transient of about 80 h as compared to the forward one of about 28 h. This is clearly attributed to the thick reinforced concrete layer close to the ambient side, which causes an even heavier heat capacity imbalance.

Based on the derived results, the forward and reverse thermal time constants $(TC)_F$ and $(TC)_R$, were calculated from (10). According to the preceding discussion it becomes clear, that the symmetrical distribution of heat capacity leads to identical values of the forward and reverse time constants, while the difference between both time constants represents the degree of the uneven distribution of heat capacity around the plane of wall symmetry. This can clearly be seen in Fig. 5 in which the derived results are appearing as discrete data points in a forward versus reverse time constant plot, the dashed diagonal line of which represents the locus of the unity ratio, $[(TC)_F/(TC)_R] = 1$. Although the data points for the symmetrical walls 4, 5 and 10 are lying on the unity ratio diagonal line, the wall 3, 6, 8 and 9 data points are at the left side of this line, owing to the larger distribution of heat capacity at the ambient side of the walls. The point 9 is at its extreme left of the same line because of the heavy heat capacity imbalance caused by the thick reinforced concrete layer close to the ambient side of the wall. For the same reason the 1, 2 and 7 data points are lying at the opposite side of the dashed line, because of the larger distribution of heat capacity at the room side of the walls.

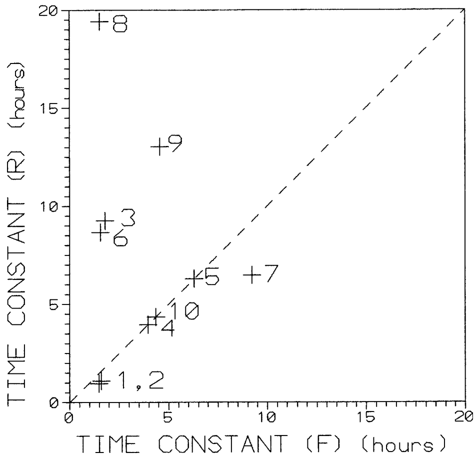

Fig. 5. Calculated forward versus reverse time constant values for the walls 1–10. For symmetrical walls the data points are lying on the diagonal broken line since $[(\mathrm{TC})_F/(\mathrm{TC})_R] = 1$.

Along with the derived forward and reverse time constants, the time required for the development of forward and reverse steady-state heat fluxes T_F^* and T_R^* respectively, was also calculated. For this purpose it was assumed that steady-state occurs as soon as the calculated heat fluxes are within 1% of the corresponding values derived by the simple thermal equilibrium considerations. The results are shown in Table 2, in which the forward and reverse time constants $(\mathrm{TC})_F$ and $(\mathrm{TC})_R$, as well as the transient duration time in hours T_F^* and T_R^* respectively, are comparatively shown for the walls 1–10.

5. The effect of thermal time constant on the transient thermal behaviour of walls

The thermal time constant is an important wall characteristic, which determines the transient thermal behaviour of the entire building shell, the energy demand and its peaking load characteristics, especially when it is intermittently heated or cooled. The effect of this quantity on the fundamental physical behaviour of walls as basic structural elements can be readily demonstrated by investigating the response of a initially isothermal wall under the influence of a rectangular temperature pulse.

For convenience, reference is made to an idealized wall, 0.28 m thick, which exchanges heat with the surrounding air by convection, with equal convective heat transfer coefficients at both sides, $h_i = h_o = 12$ W/m^2 K. The wall is made of concrete and it is insulated by a 4 cm thick thermal insulation layer. This layer is allowed to move towards x axis at a direction perpendicular to the wall plane in 4 cm steps, from the ambient side of the wall ($x = 0$) to the corresponding room side ($x = 25$ cm), so as the concrete layer always be fixed at 24 cm thick. Assuming that the thermal insulation layer moves towards a direction normal to the wall surface at the fixed distances of $x = 0$, 5.0, 9.0, 13.0, 17.0, 21.0 and 25.0 cm, a sequence of seven fixed thermal resistance composite walls of decreasing forward and increasing reverse time constants will be developed.

Forward thermal time constants of 8.85, 7.95, 6.83, 5.42, 3.75, 1.96 and 0.16 h were calculated for the seven walls, which will be assigned as W1–W7 respectively. The results were derived under the assumption of an initial wall temperature of 5 °C equal to the ambient temperature, for a step change of room air temperature of 20 °C. It can be seen that the forward time constant decreases drastically as the thermal insulation layer moves towards x axis, so as a minimum value as low as 0.16 h is derived when the free surface of the thermal insulation layer is directly exposed to the room air. When the ambient air undergoes the same step temperature change while the room air is maintained at the fixed temperature of 5 °C, the corresponding calculated reverse thermal time constants for the walls W1–W7 were found to be 0.16, 1.96, 3.75, 5.42, 6.83, 7.95 and 8.85 h respectively. Both forward and reverse time constants for $x = 13$ cm are identical $[(\mathrm{TC})_F/(\mathrm{TC})_R] = 1$, and equal to 5.42 h owing to the full symmetry of the wall.

The implication of the varying thermal time constant of the group of seven uniform thermal resistance walls is demonstrated by investigating the effect of a rectangular temperature pulse of $T_r(t) = 20$ °C at $t = t_o$. The pulse is applied for 24 h at the room side of the walls, which are isothermal at an initial temperature of 5 °C equal to the ambient temperature. Transient heat fluxes are developed, which gradually converge to the corresponding steady-state solutions as can be seen in Fig. 6, in which the solid line curves correspond to the time-depended heat fluxes $q(t)$, for the walls W1–W7 throughout the whole simulation time domain of 120 h.

The transient response of the wall W7 with the minimum forward time constant of $(\mathrm{TC})_F = 0.16$ h, leads to an immediate development of a short negative heat flux pulse of a maximum absolute value of 28.8 W/m^2, which rapidly decreases and approaches its equilibrium heat flux of 11.68 W/m^2. The same immediate response is exhibited at $t = 24$ h with the development of a positive maximum heat flux of 16.7 W/m^2, which rapidly approaches the corresponding steady-state zero value.

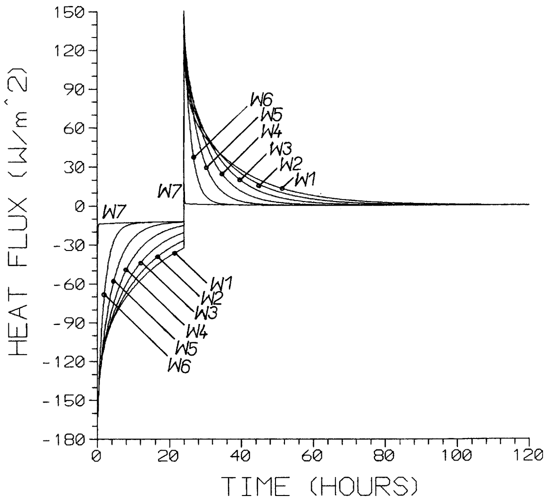

Fig. 6. Calculated transient heat fluxes for the isothermal walls W1–W7 under the influence of a rectangular pulse of air temperature of 20 °C for a period of 24 h at the room side, when the ambient air is at a fixed temperature of 5 °C.

The sequence of the next six solid lines of decreasing slope in Fig. 6 corresponding to the walls W6–W1, represent the effect of the increasing thermal time constant from 1.96 to 8.85 h on the wall transient thermal behaviour. Large negative heat fluxes of a maximum absolute value of 162 W/m^2 are developed at $t = t_o$, which decrease with time at a rate which is inversely proportional to the forward wall time constant. It can be seen that the transient heat flux becomes equal to its corresponding steady-state value of 11.68 W/m^2 for the walls W6 and W5 within 24 h, while this lasts appreciably longer as the time constant increases to its maximum value of 8.85 h for the wall W1. Corresponding maximum heat fluxes between 150 and 130 W/m^2 are developed at $t = 24$ h for the walls W6–W1 respectively, which decrease at a rate inversely proportional to the wall time constant.

The energy flow at the room side of the wall during the period of $T^* = 24$ h is calculated as,

$$E_{\mathrm{r,24\,h}} = \int_0^{T^*} q(L,t)\mathrm{d}t = q_\mathrm{s} \cdot T^* + \int_0^{T^*} [q(L,t) - q_\mathrm{s}]\mathrm{d}t \tag{12}$$

which is represented by the area underneath the transient heat flux curves. The first additive term in the above right hand side expression represents the energy lost through the wall while the second term is the stored energy during $T^* > t > 0$, which is represented by the surface area between curves and the horizontal line of $q_\mathrm{s} = 11.68$ W/m^2. This energy which is delivered back to the room for $\infty > t > T^*$, is considered as a heat gain and it was found to be numerically equal to the energy underneath heat flux curves for the same period of time, so as

$$\int_0^{T^*} [q(L,t) - q_\mathrm{s}]\mathrm{d}t = \int_{T^*}^{\infty} q(L,t)\mathrm{d}t \tag{13}$$

This means that for occasionally heated or cooled spaces and constant or slowly varying ambient temperature, the walls of low forward time constant are more suitable, otherwise a substantial amount of stored energy will be available as energy gain to the space at a time period of no energy demand. They will also contribute to the leveling out of the peak load demand, since substantially lower peak heat fluxes will be developed at $t = 0$ and 24 h. These effects are becoming less significant as the rectangular pulse period becomes longer and especially for continuously heated or cooled spaces. When short temperature changes occur at the ambient side and especially when they are combined with simultaneous solar radiation effects, for the same reason the walls of lower reverse time constants should be considered as more suitable. When the period of temperature changes at both wall sides is unpredictable or even stochastic, walls of comparable forward and reverse time constants like W3–W5 may be rather more preferable.

6. Conclusions

The walls are vital elements of the building shell, and their transient thermal behaviour strongly influences the thermal characteristics of the entire building. Depending on the specific structure, a building shell is usually composed of curtain walls or a combination with support frames. A quantity which is related to the dynamic thermal behaviour of the building shell is the thermal time constant, which physically reflects how rapidly the wall responds to a given step temperature change of air, at a specific wall side. This parameter can also be considered as a significant physical quantity for wall taxonomy. For specified initial and boundary conditions, it depends on the mass and specific heat capacity distribution of the wall layers in respect to the given wall side. In the case of symmetrical walls the thermal response is identical in relationship to both wall boundaries.

An analysis was developed for the calculation of the wall thermal time constant and the derived results were comparably presented for a wide range of usual wall designs, currently employed in construction practice. Since this physical quantity is assigned to a specific wall side, unlikely to the U value or thermal resistance of a composite wall, two different thermal time constants, the forward and reverse were defined. It was found that although they may usually differ, depending on the spatial distribution of individual layer heat capacity, they become equal for symmetrical walls, owing to their identical thermal response to a step air temperature change at both wall sides.

The significance of this quantity was demonstrated by a reference to the effects caused by a rectangular pulse of air temperature to a group of fixed thermal resistance walls, of growing thermal time constant. Among other interesting issues relating this parameter to the fundamental physical behaviour of buildings, it should be worthwhile to note the advantage of employing low thermal forward time constant walls in intermittently heated or cooled and scarcely occupied spaces, something which contributes to the improved thermal design of structures.

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